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GALVANOMAGNETIC MEASUREMENTS TO DISTINGUISH AMONG DIFFERENT ENVIRONMENTS OR TO ASCERTAIN ANISOTROPIC BEHAVIOUR INDEPENDENT FROM THE CRYSTAL SYMMETRY

By

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Abstract: *In this work a brief description of the traditional galvanomagnetic measurements in the weak field case is presented and also emphasis is given to the importance and the use of the Wasscher's method for the detection of the anisotropic electric behaviour which is not caused by the crystal symmetry of the material. In continuation it is described how it is possible to distinguish cubic from non-cubic environments in the case of (001), (110) and (111) oriented thin films and surface layers. Finally experimental results deduced from the measurements on mechanically deformed (111)-sample of monocrystalline Ge are given. So anisotropy was observed on the zero field electric resistivity with principal directions the axes [$1\bar{1}0$] and [$11\bar{2}$]. Also the magnetoresistance skewness effect was observed by changing the direction of the magnetic field parallel to the sample plane. With the magnetic field perpendicular to the sample plane also an anisotropic magnetoresistance was observed. Therefore six different magnetoresistance coefficients were determined showing that the "symmetry" of the anisotropy is much lower than the cubic one on the (111) plane.*

1. INTRODUCTION

We use to say that the characteristic properties of a material (optical, electric) depend on its crystalline symmetry. Therefore the crystal class expresses the isotropic or anisotropic behaviour of the material. In the case of the galvanomagnetic (GVM) effects the correlation between the electromagnetic magnitudes and the crystalline symmetry is expressed by the non-zero tensor elements of the zero-field resistivity ρ_{ij}, the weak-field Hall coefficient ρ_{ijk} and the weak-field magnetoresistance (WFMR) coefficients ρ_{ijkl}.

Until now it has been assumed that the symmetry of the experimental results is normally associated with the material under study. But is that true in the reality? Or are there other factors too whose influence changes the situation? And in what way is it possible to distinguish the existence of a such different situation?

The answer is that we must determine the most of the previous tensor elements if we want to have a clear picture of the reality. But for the measurement of all the GVM coefficients we need more than one sample (see for example the case of Bi_2Te_3 [1, 2]), and additionally the acceptance that there are no alterations from one sample to the other.

In the case e.g. of cubic crystals, most WFMR measurements are made on (001) samples with **J** in the directions [100] or [110]. Samples parallel to (111) plane have been used much less. Also it has been traditional to made no more than three different WFMR measurements on a sample because a fourth measurement is impossible or unnecessary.

Under weak-field conditions the relation between **E**, **J** and **B** is expressed by the equation

$$E_i = \rho_{ij}J_j + \rho_{ijk}J_jB_k + \rho_{ijkl}J_jB_kB_l \tag{1}$$

In the case of cubic symmetry eq. (1) takes the Seitz-Pearson-Suhl (SPS) [3, 4] form

$$\mathbf{E} = \rho_0\mathbf{J} + a(\mathbf{J} \times \mathbf{B}) + bB^2\mathbf{J} + c(\mathbf{J} \cdot \mathbf{B})\mathbf{B} + dT\mathbf{J} \tag{2}$$

where a, b, c, d are the SPS coefficients and T the diagonal tensor $[B_i^2]$.

Thus we can write the magnetoresistance coefficients M [5] of the equation

$$\frac{\Delta\rho}{\rho_0} = M(\mu_H B)^2 \tag{3}$$

where μ_H is the Hall mobility
as

$$M = b + c(\sum_s i_s n_s)^2 + d\sum_s i_s^2 n_s^2 \tag{4}$$

where i_s and n_s are the direction cosines of **J** and **B** respectively.

Allgaier et al. [6-9] have shown that eq. (4) can be used to give four independent magnetoresistance measurements from only one sample. In this way it is possible to distinguish one crystalline symmetry from another one as in the case of (001) and (111) oriented thin films or surface layers with cubic or non-cubic symmetry.

It should be noted here that the zero-field resistivity on the plane of the sample remains isotropic even in the case of lower symmetry while the magnetoresistance anisotropy is again associated with the crystalline symmetry of the environment. Also this method can't give simple expressions of the eq. (4) in all the cases and so it is impossible to use it for other planes of samples like in the case of the (110) plane.

As the symmetry of the material is lowered expression (1) can no longer be reduced to a simple SPS-type formula because the number of distinct non-zero coefficients increases considerably, and because the zero-field resistivity is, in general, no longer isotropic and of course with this classical configuration we can't distinguish the situations which are not caused by crystalline symmetry.

For these reasons we need a more convenient method for measurements and this is achieved by Van der Pauw-Wasscher technique [10, 11]. In order to get a measurement with this method we use a

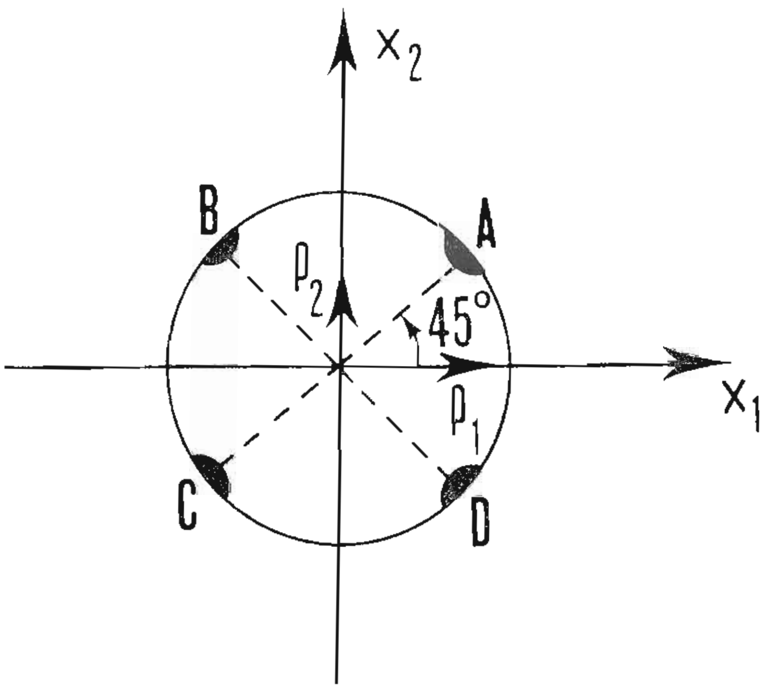

Fig. 1. Planar anisotropic circular sample with contacts in the right position for the determination of ϱ_1 and ϱ_2.

planar, circular sample with four contacts ABCD taken along the circumference on two perpedicular diametres (Fig. 1). We may then define the "resistances".

$$R_1 = \frac{V_D - V_C}{I_{AB}}, \quad R_2 = \frac{V_A - V_D}{I_{BC}} \quad \text{and} \quad R_{12} = \frac{V_A - V_C}{I_{BD}}. \quad (5)$$

In the case of an isotropic material with this configuration we have $R_1 = R_2 = R$ and the zero field resistivity is given by the equation

$$\rho = \frac{\pi d}{\ln 2} R$$

where d is the thickness of the sample.

In the case of an anisotropic material the maximum value of the resistance ratio

$$(R_1/R_2)_{max} = (R_1)_{max} \,/\, (R_2)_{min}$$

is obtained when the contacts are placed at an angle of 45° to the directions of the principal axes of resistivity x_1 and x_2. So for the main values of resistivity we have the expressions

$$\rho_1 = \frac{\lambda^{1/2}\pi d (R_1)_{max}}{\ln \frac{2}{1-k}} \quad (7)$$

$$\rho_2 = \frac{\lambda^{-1/2}\pi d (R_2)_{min}}{\ln \frac{2}{1+k}} \quad (8)$$

where λ is the anisotropic ratio ρ_1/ρ_2 ($\rho_1 > \rho_2$) and k the modulus of an elliptic integral which is related to the ratio λ. Thus with this method we have the two principal resistivities from one measurement and of course the resistivity in any direction on the plane of the sample by the well-known equation [12].

$$\rho = \rho_1 \cos^2\omega + \rho_2 \sin^2\omega \quad (9)$$

where ω is the angle between the directions of ρ_1 and ρ.

The Hall coefficient is determined from the relation

$$R_H = \frac{d}{B} \Delta R_{12} \tag{10}$$

where B is the perpendicular magnetic field and ΔR_{12} the change in the resistance R_{12} due to the field **B**.

It should be noted here that the method is very sensitive for the ascertainment of the electric anisotropy. This is evident from Fig. 2

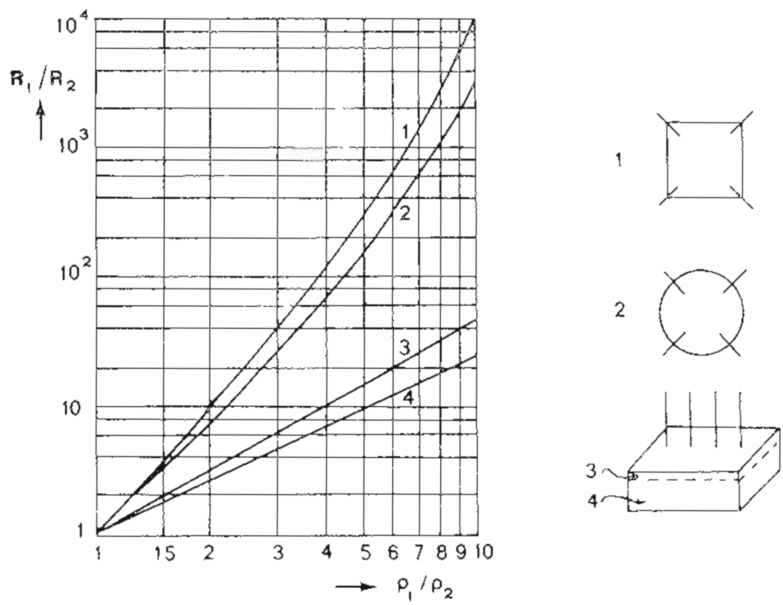

Fig. 2. The dependence of the ratio $(R_1/R_2)_{max}$ upon the main resistance ratio $\lambda = \varrho_1/\varrho_2$ for various sample configurations.

[11] which indicates that at a value λ, corresponds a greater ratio $(R_1/R_2)_{max}$.

The influence of the magnetic field is also of a great importance because it causes changes in the direction of motion of the charge carriers. Thus the magnetic field introduces an anisotropy in the electric resistivity which competes the physical one. The final situation is an anisotropy which in general has principal directions different from those of the zero-field resistivity. This phenomenon is called magnetoresistance skewness [13] and it is shown in Fig. 3. If the direction cosines

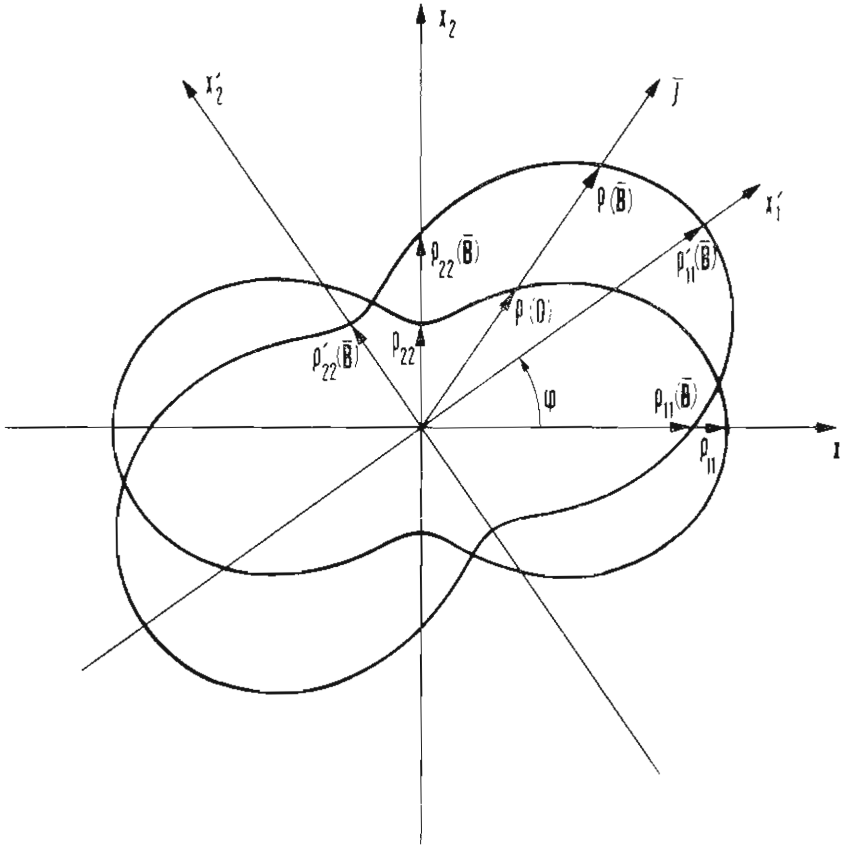

Fig. 3. Polar plots of the anisotropic resistivity in the sample plane with and without the presence of the magnetic field **B**.

of the magnetic field (in respect to the x_i axes) are u, v, o then the main resistivities in the new system axes x_i' are [14]

$$\rho'_{11}(\mathbf{B}) = \frac{1}{2}[\rho_{11}(\mathbf{B}) + \rho_{22}(\mathbf{B})] + \frac{\rho_{12}(\mathbf{B})}{\sin 2\varphi} \tag{11}$$

$$\rho'_{22}(\mathbf{B}) = \frac{1}{2}[\rho_{11}(\mathbf{B}) + \rho_{22}(\mathbf{B})] - \frac{\rho_{12}(\mathbf{B})}{\sin 2\varphi} \tag{12}$$

while the angle of skewness φ is given by the equation

$$\tan 2\varphi = \frac{2\rho_{12}(\mathbf{B})}{\rho_{11}(\mathbf{B}) - \rho_{22}(\mathbf{B})} \tag{13}$$

So the experimental measurement of the angle φ leads to the determination of the WFMR coefficient of the type ρ_{ijij} $(i \neq j)$.

In the following we shall see how all the above could be applicable to distinguish among different environments or to ascertain anisotropic behaviour independent from the crystal symmetry.

2. MEASUREMENTS TO DISTINGUISH AMONG DIFFERENT ENVIRONMENTS

In the case of cubic crystals there are five different GVM coefficients namely

$$\rho_0,\ \rho_{123},\ \rho_{1111},\ \rho_{1122} \text{ and } \rho_{1212} \tag{14}$$

Thus the zero-field resistivity and the weak field Hall effect are isotropic, while there are three WFMR coefficients which are related to the types of band structures and scattering anisotropies.

If the direction cosines of **J** and **B** are p, q, r and u, v, w respectively then the zero field resistivity in the direction of **J** is

$$\rho(0) = \frac{\mathbf{E} \cdot \mathbf{J}}{J^2} = \rho_0(p^2 + p^2 + r^2) = \rho_0 \tag{15}$$

while in the presence of the magnetic field B we have

$$\begin{aligned} \rho(\mathbf{B}) = \frac{\mathbf{E}(\mathbf{B}) \,.\, \mathbf{J}}{J^2} = \\ &= [\rho_0 + (\rho_{1111}u^2 + \rho_{1122}v^2 + \rho_{1122}w^2)B^2]p^2 \\ &+ [\rho_0 + (\rho_{1122}u^2 + \rho_{1111}v^2 + \rho_{1122}w^2)B^2]q^2 \\ &+ [\rho_0 + (\rho_{1122}u^2 + \rho_{1122}v^2 + \rho_{1111}w^2)B^2]r^2 \\ &+ 4\rho_{1212}B^2[uvpq + vwqr + wurp]. \end{aligned} \tag{16}$$

In the next we consider (001), (110) and (111) oriented thin films and surface layers having cubic, tetragonal and trigonal or hexagonal symmetry [15].

2.1. Sample parallel to the (001) plane

In this case the sample plane contains the two axes x_1 and x_2, and we have **J** = (Jcosω, Jsinω, 0) (Fig. 4). To obtain the GVM coefficients we perform measurements in the following manner.

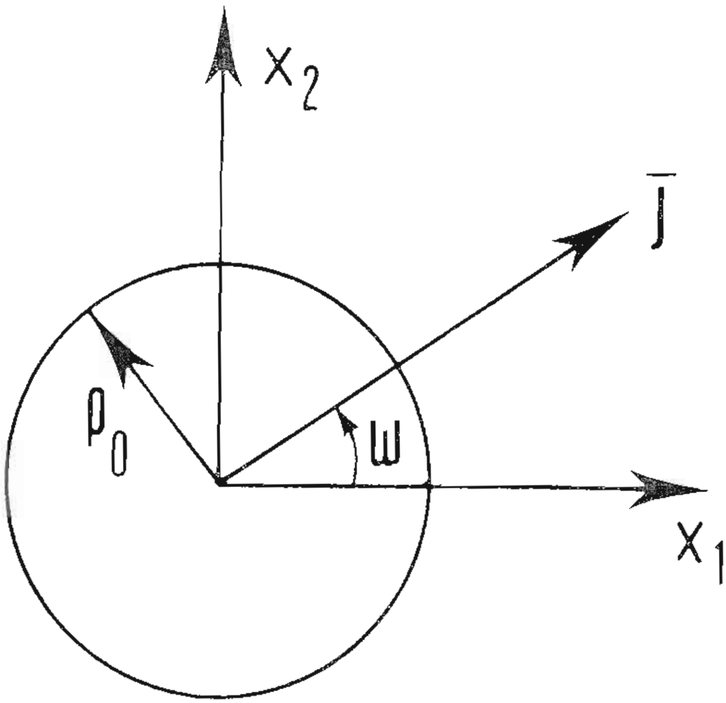

Fig. 4. Polar distribution of ρ on the (x_1, x_2) plane.

2.1.1 Specific Resistance and Hall Coefficient

The zero-field resistivity is given by eq. (6) while the Hall coefficient $R_H = \rho_{123}$ is determind by eq. (10).

2.1.2 WFMR Coefficients

Two configurations will be treated:

α) $\mathbf{B} = (B, 0, 0)$. Using Figure 4 and eq. (16) we obtain

$$\rho(\mathbf{B}) = (\rho_0 + \rho_{1111}B^2)\cos^2\omega + (\rho_0 + \rho_{1122}B^2)\sin^2\omega \qquad (17)$$

corresponding to a resistivity anisotropy with principal axes lying in the crystallographic directions x_1 and x_2 (Fig. 5). By measuring the experimental quantities ρ_0, $\rho_{11}(\mathbf{B})$ and $\rho_{22}(\mathbf{B})$ we obtain

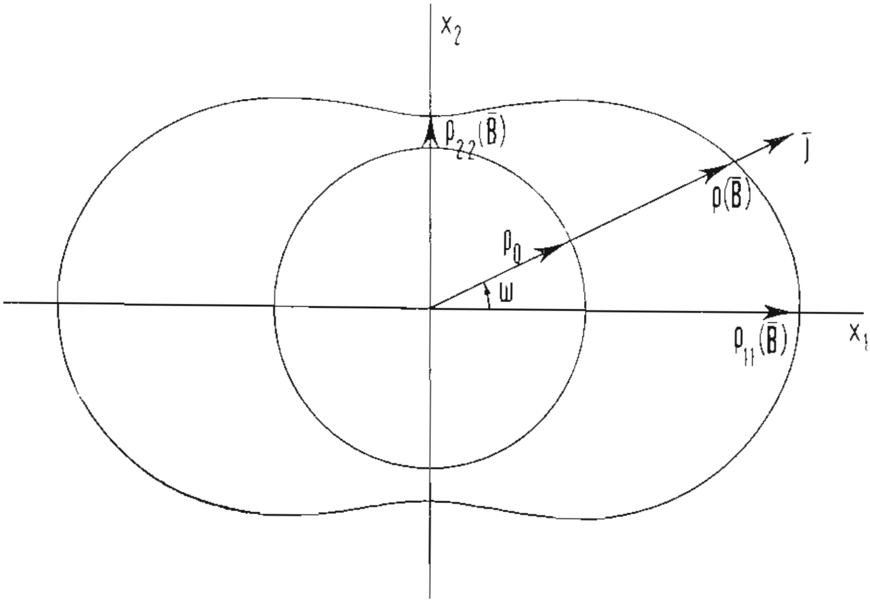

Fig. 5. The anisotropy in resistanee induced by the magnetic field.

$$\Delta\rho = \rho_{1111}B^2\cos^2\omega + \rho_{1122}B^2\sin^2\omega$$

$$= \Delta\rho_{11}\cos^2\omega + \Delta\rho_{22}\sin^2\omega \qquad (18)$$

where

$$\Delta\rho_{11} = \rho_{11}(\mathbf{B}) - \rho_0, \qquad \Delta\rho_{22} = \rho_{22}(\mathbf{B}) - \rho_0 \qquad (19)$$

Thus we are led to the relations

$$\rho_{1111} = \frac{\Delta\rho_{11}}{B^2} \quad \text{and} \quad \rho_{1122} = \frac{\Delta\rho_{22}}{B^2} \qquad (20)$$

by which ρ_{1111} and ρ_{1122} may be determined from experimental measurements of $\Delta\rho_{11}$ and $\Delta\rho_{22}$. It is evident that with the above direction of **B**, $\Delta\rho_{11}$ corresponds to the longitudinal magnetoresistance while $\Delta\rho_{22}$ corresponds to the transverse one. It should be noted here that in the case of B = (0, B, 0), the same anisotropy appears with a phase difference of 90° so that the same two WFMR coefficients, ρ_{1111} and ρ_{1122}, are obtained, but in reverse order. Finally for **B** = (0, 0, B) the sample remains isotropic and only the coefficient ρ_{1122} can be determined.

This last case is interesting when we have (001) oriented films or surface layers. Suppose that the value of ρ_{1122} as determined from this configuration is different from that determined by eq. (20). This result is an indication of tetragonal, rather than cubic symmetry, with the tetragonal axis lying along the [001] direction.

b) **B** = (Bu, Bv, 0). In this case an anisotropic (skewed) behaviour is expected, such that the principal axes and the crystallographic axes lie in different directions (Fig. 6). If these directions are at an angle φ to each other then the following equation is valid

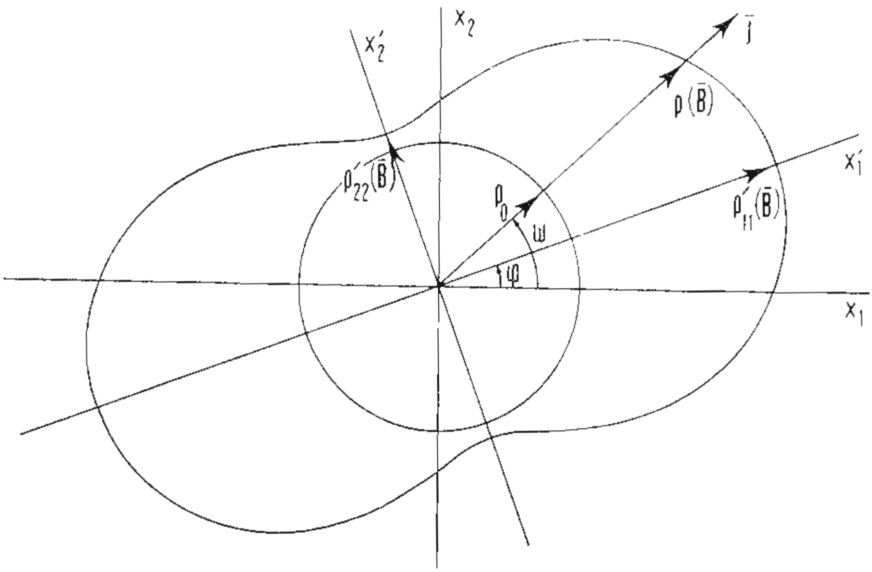

Fig. 6. Determination of Δϱ for anisotropy that has its principal axes inclined by an angle φ to the crystallographic axes.

$$4\rho_{1212}B^2uv = \sin 2\varphi[\rho'_{11}(\mathbf{B}) - \rho'_{22}(\mathbf{B})] \tag{21}$$

where $\rho'_{11}(\mathbf{B})$, $\rho'_{22}(\mathbf{B})$ are the principal resistivities, under the influence of the magnetic field. In this way, the coefficient ρ_{1212} is obtained. The procedure is to choose first a certain direction of **B** with respect to the (x_1, x_2) set of axes and then rotate the system of contacts, without changing the direction of **B**, until the maximum value of R_1/R_2 is found; thus the value of the angle φ and the resistances $\rho'_{11}(\mathbf{B})$ and $\rho'_{22}(\mathbf{B})$ are obtained.

2.2. Sample parallel to the (110) plane

In the case that the specimen is parallel to a (110) plane as in the case of thin films grown on (110) substrates, a new set of axes should be chosen in terms of which the measurements may be defined. We use the coordinate system $[1\bar{1}0] = x_1'$, $[110] = x_2'$ and $[001] = x_3'$ (Fig. 7). With this set, the plane of the sample contains the x_1' and

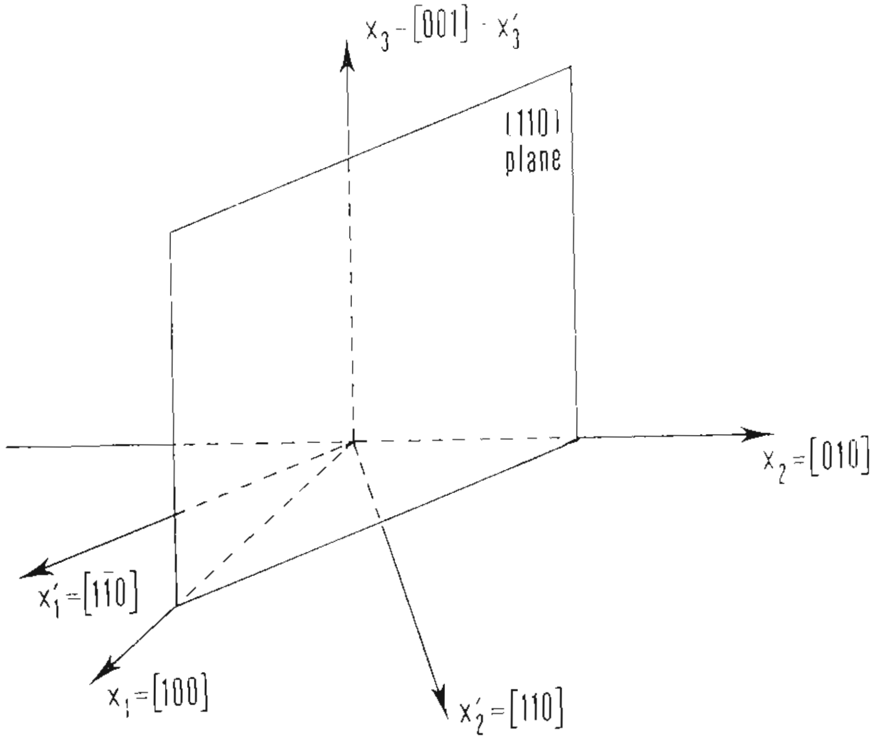

Fig. 7. The coordinate system in the case of a sample parallel to the (110) plane.

x_3' axes. Axes x_1' and x_2' exhibit two-fold, and x_3' fonr-fold rotational symmetry. These axes are similar to the axes of the tetragonal system for the groups $D_4(422)$, $C_{4v}(4mm)$, D_{2d}, $V_d(\bar{4}2m)$ and $D_{4h}(4/mmm)$.

The resistivity and Hall coefficients for these groups involve the four different nonzero tensor elements (16).

$$\rho_{11}(=\rho_{22})\,, \quad \rho_{33}\,, \quad \rho_{123} \quad \text{and} \quad \rho_{231}(=\rho_{312}) \tag{22}$$

while the nonzero WFMR coefficients are presented in table I, using the following correspondence:

$$11 \longrightarrow 1 \qquad 22 \longrightarrow 2 \qquad 33 \longrightarrow 3$$

$$23 = 32 \longrightarrow 4 \qquad 31 = 13 \longrightarrow 5 \qquad 12 = 21 \longrightarrow 6.$$

TABLE I

The non-zero WFMR coefficients for the groups D_4, C_{4v}, D_{2d}, V_d, D_{4h}.

	B_1^2	B_2^2	B_3^2	B_2B_3	B_3B_1	B_1B_2
E_1, J_1	ρ_{11}	ρ_{12}	ρ_{13}	0	0	0
E_2, J_2	ρ_{12}	ρ_{11}	ρ_{13}	0	0	0
E_3, J_3	ρ_{31}	ρ_{31}	ρ_{33}	0	0	0
E_2, J_3	0	0	0	$2\rho_{44}$	0	0
E_3, J_1	0	0	0	0	$2\rho_{44}$	0
E_1, J_2	0	0	0	0	0	$2\rho_{66}$

In the case of cubic symmetry, we are seeking, by the transformation of the elements from the x_i to the new set, the CVM coefficients presented in eq. (22) and in table I. Thus we have the following results

$$\rho'_{11} = \rho'_{22} = \rho'_{33} = \rho_0 \tag{23}$$

$$\rho'_{123} = \rho'_{231} = \rho'_{312} = \rho_{123} \tag{24}$$

$$\rho'_{1111} = \rho'_{2222} = \frac{1}{2}(\rho_{1111} + \rho_{1122} + 2\rho_{1212}) \tag{25}$$

$$\rho'_{3333} = \rho_{1111} \tag{26}$$

$$\rho'_{1133} = \rho'_{2233} = \rho'_{3311} = \rho'_{3322} = \rho_{1122} \tag{27}$$

$$\rho'_{2211} = \rho'_{1122} = \frac{1}{2}(\rho_{1111} + \rho_{1122} - 2\rho_{1212}) \tag{28}$$

$$\rho'_{1212} = \frac{1}{2}(\rho_{1111} + \rho_{1122}) \tag{29}$$

$$\rho'_{2323} = \rho'_{1313} = \rho_{1212} \tag{30}$$

The difference between the cubic case, eqs. (23) — (30), and tetragonal symmetry, eq. (22) and table I, is obvious. The problem is thus reduced to the determination of the $\rho_{ij}(\mathbf{B})$ from measurements of the $\rho'_{ki}(\mathbf{B})$, thereby to the deduction of the tetragonal environment in case of reduced symmetry. The procedure is as follows:

2.2.1. Specific resistance and Hall coefficient.

Using the method presented in 1, we measure $\rho'_{11} = \rho'_{33} = \rho_0$ and $\rho'_{123} = \rho_{123}$. In case the actual symmetry is tetragonal, however, $\rho'_{11} \neq \rho'_{33}$. Thus by rotating the four contacts, a maximum in the ratio R_1/R_2 can be found, thus determining the anisotropy ratio λ. From eqs. (7) and (8) ρ'_{11} and ρ'_{33} are determined and the directions of the x'_1, x'_3 axes are defined. In this manner, the reduced symmetry is revealed and evaluated. It should also be mentioned that the measurement of the Hall coefficient with perpendicular magnetic field [$\mathbf{B} = (0, B, 0)$] yields the coefficient $\rho'_{231} = \rho'_{312}$.

2.2.2. WFMR Coefficients.

In the case of $\mathbf{B} = (Bu, Bv, Bw)$ and $\mathbf{J} = (Jp, 0, Jr)$ (referred to the primed coordinate system), the anisotropy that appears has principal axes which, in general, are different from the x_1' and x_3' axes. For cubic symmetry, the measurement of ρ'_{1122}, ρ'_{3311} $(= \rho'_{1133})$, and ρ'_{3333} alows the determination of ρ_{1111}, ρ_{1122}, and ρ_{1212} from the relations (26), (27) and (28). Thus

a) For $\mathbf{B}(0, B, 0)$ (that is, perpendicular to the sample plane) we have

$$\rho(\mathbf{B}) = [\rho'_{11} + \rho'_{1122}B^2]p^2 + [\rho'_{33} + \rho'_{3311}B^2]r^2 \tag{31}$$

and

$$\Delta\rho = \rho'_{1122}B^2p^2 + \rho'_{3311}B^2r^2 = \Delta\rho'_{11}p^2 + \Delta\rho'_{33}r^2, \tag{32}$$

where

$$\Delta\rho'_{11} = \rho'_{11}(\mathbf{B}) - \rho'_{11}, \qquad \Delta\rho'_{33} = \rho'_{33}(\mathbf{B}) - \rho'_{33}. \tag{33}$$

This leads to

$$\rho'_{1122} = \frac{\Delta\rho'_{11}}{B^2} \text{ and } \rho'_{3311} = \frac{\Delta\rho'_{33}}{B^2} \tag{34}$$

b) For $\mathbf{B}(0, 0, B)$ we have

$$\rho(\mathbf{B}) = [\rho'_{11} + \rho'_{1133}B^2]p^2 + [\rho'_{33} + \rho'_{3333}B^2]r^2 \quad (35)$$

which leads to

$$\rho'_{1133} = \frac{\Delta\rho'_{11}}{B^2} \text{ and } \rho'_{3333} = \frac{\Delta\rho'_{33}}{B^2} \quad (36)$$

The case of reduced, tetragonal symmetry is immediately apparent from these measurements if we find that

$$\rho'_{3311} \neq \rho'_{1133} \quad (37)$$

2.3. Sample parallel to the (111) plane.

In many instances we deal with material that cleaves along the

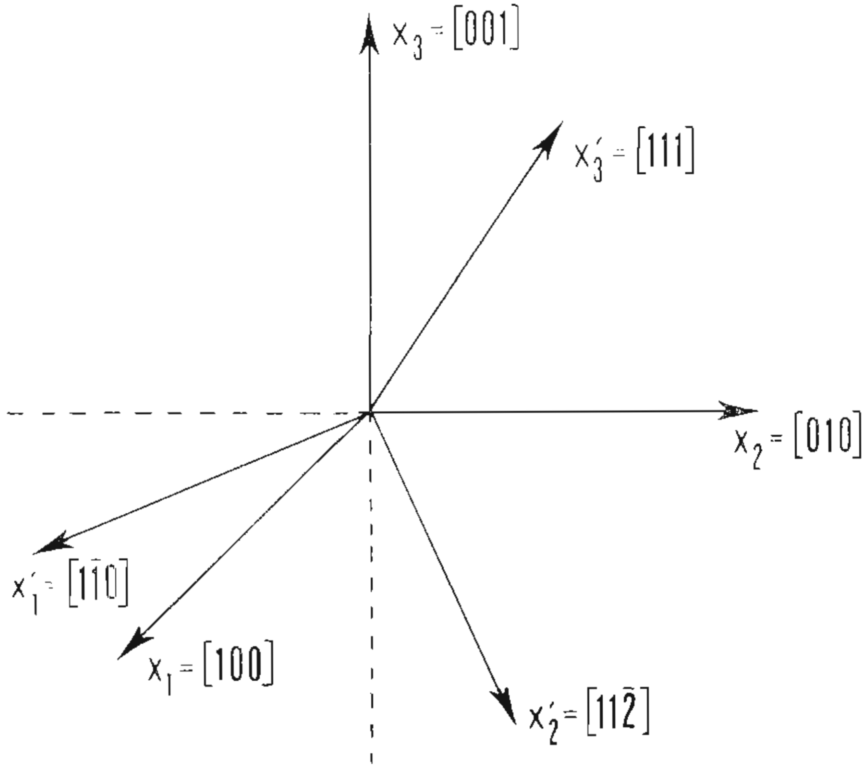

Fig. 8. The coordinate system in the case of a sample parallel to the (111) plane.

(111) plane. As in the previous section, we should again choose a sample-oriented coordinate system in terms of which the tensor elements are more conveniently defined and measured. As a new frame of reference we choose the axes $[1\bar{1}0] = x_1'$ and $[11\bar{2}] = x_2'$ in the (111) plane and the $[111] = x_3'$ perpendicular to the (111) plane (Fig. 8). The x_1', x_2' and x_3' directions coincide with the two-fold axis, the disectrix and the three-fold axis, respectively, of trigonal crystals [C_{3v}(3m) and D_{3d}(3m) groups].

The non-zero elements of the resistivity and Hall tensors for this group are [16, 17].

$$\rho_{11}(=\rho_{22}), \quad \rho_{33}, \quad \rho_{123} \quad \text{and} \quad \rho_{231}(=\rho_{312}) \tag{38}$$

TABLE II

The non-zero WFMR coefficients for the groups C_{3v} and D_{3d}.

	B_1^2	B_2^2	B_3^2	B_2B_3	B_3B_1	B_1B_2
E_1, J_1	ρ_{11}	ρ_{12}	ρ_{13}	$2\rho_{14}$	0	0
E_2, J_2	ρ_{12}	ρ_{11}	ρ_{13}	$2\rho_{14}$	0	0
E_3, J_3	ρ_{31}	ρ_{31}	ρ_{33}	0	0	0
E_2, J_3	ρ_{41}	ρ_{41}	0	$2\rho_{44}$	0	0
E_3, J_1	0	0	0	0	$2\rho_{44}$	$2\rho_{41}$
E_1, J_2	0	0	0	0	$2\rho_{14}$	$\rho_{11} - \rho_{12}$

while the non-zero WFMR coefficients are given in table II with the following corespondence:

$$11 \longrightarrow 1 \qquad 22 \longrightarrow 2 \qquad 33 \longrightarrow 3 \qquad 23 = 32 \longrightarrow 4$$

$$13 = 31 \longrightarrow 5 \qquad 12 = 21 \longrightarrow 6$$

Returning now to the cubic case. With the transformation from the x_i' coordinates we are seeking the GVM coefficients of eq. (38) and table II. Thus we have the following results

$$\rho'_{11} = \rho'_{22} = \rho'_{33} = \rho_0 \tag{39}$$

$$\rho'_{123} = \rho'_{231} = \rho'_{312} = \rho_{123} \tag{40}$$

$$\rho'_{1111} = \rho_{2222} = \frac{1}{2}(\rho_{1111} + \rho_{1122} + 2\rho_{1212}) \tag{41}$$

$$\rho'_{3333} = \frac{1}{3}(\rho_{1111} + 2\rho_{1122} + 4\rho_{1212}) \tag{42}$$

$$\rho'_{1122} = \rho'_{2211} = \frac{1}{6}(\rho_{1111} + 5\rho_{1122} - 2\rho_{1212}) \tag{43}$$

$$\rho'_{1133} = \rho'_{3311} = \rho'_{2233} = \rho'_{3322}$$

$$= \frac{1}{3}(\rho_{1111} + 2\rho_{1122} - 2\rho_{1212}) \tag{44}$$

$$\rho'_{2323} = \rho'_{3131} = \frac{1}{3}(\rho_{1111} - \rho_{1122} + \rho_{1212}) \tag{45}$$

$$\rho'_{1123} = -\rho'_{2223} = \rho'_{1231} = \rho'_{2311} = -\rho'_{2322}$$

$$= \rho'_{3112} = \frac{2}{6}(\rho_{1111} - \rho_{1122} - 2\rho_{1212}) \tag{46}$$

and

$$\rho'_{1212} = \frac{1}{2}(\rho'_{1111} - \rho'_{1122}). \tag{47}$$

The difference between cubic and trigonal symmetry is obvious from a comparison of eqs. (39)-(47) with eq. (38) and table II. The problem is again reduced to the determination of the $\rho_{ij}(\mathbf{B})$ by measuring the $\rho'_{ki}(\mathbf{B})$ and, in the case of reduced symmetry, of determining the trigonal environment. The procedure to be followed is outlined below.

2.3.1. Specific resistance and Hall coefficient

By using the method presented in 1, we measure the $\rho'_{11} = \rho'_{22} = \rho_0$ and $\rho'_{123} = \rho_{123}$. If there is an environment of trigonal symmetry it cannot be detected here, since on a specimen with faces parallel to the (x_1, x_2) [= (111)] plane, the measurements of ρ'_{33} and ρ'_{231}, are impossible.

2.3.2. WFMR Coefficients

If $\mathbf{B} = (Bu, Bv, Bw)$ and $\mathbf{J} = (Jp, Jq, 0)$ (in the primed system of reference), the resistivity anisotropy will appear with the principal axes different from the x_1' and x_2' directions. For cubic symmetry the measurement of ρ'_{1111}, ρ'_{1122}, and ρ'_{1133} allows the determination of ρ_{1111}, ρ_{1122}, and ρ_{1212} from the relations (41), (43) and (44). Now we consider the specific cases.

a) $\mathbf{B} = (B, 0, 0)$. Then

$$\rho(\mathbf{B}) = (\rho'_{11} + \rho'_{1111}B^2)p^2 + (\rho'_{22} + \rho'_{1122}B^2)q^2; \tag{48}$$

that is, the anisotropy axes are x_1' and x_2', so that

$$\Delta\rho = \rho'_{1111}B^2p^2 + \rho'_{1122}B^2q^2 = \Delta\rho'_{11}p^2 + \Delta\rho_{22}'q^2, \tag{49}$$

where

$$\Delta\rho'_{11} = \rho'_{11}(B) - \rho'_{11} \quad \text{and} \quad \Delta\rho'_{22} = \rho'_{22}(\mathbf{B}) - \rho'_{22} \tag{50}$$

Therefore

$$\rho'_{1111} = \frac{\Delta\rho'_{11}}{B^2} \text{ and } \rho'_{1122} = \frac{\Delta\rho'_{22}}{B^2} \tag{51}$$

b) $B = (0, 0, B)$. Then

$$\rho(\mathbf{B}) = (\rho'_{11} + \rho'_{1133}B^2)p^2 + (\rho'_{22} + \rho'_{1133}B^2)q^2 = \rho'_{11} + \rho'_{1133}B^2, \tag{52}$$

which means that this case is isotropic $(\rho'_{11} = \rho'_{22})$.

Therefore
$$\rho'_{1133} = \frac{\rho(\mathbf{B}) - \rho'_{11}}{B^2} \tag{53}$$

c) If the symmetry is cubic, then six different magnitudes are involved in the WFMR coefficients, eqs. (41) to (46) (only three are independent), while in the case of trigonal sxmmetry eight become independent, with the new inequalities $\rho'_{3311} \neq \rho'_{1133}$ and $\rho'_{2311} \neq \rho'_{1123}$. Therefore by calculating a fourth coefficient, e.g. the ρ'_{1123} from the relation (46), using ρ_{1111}, ρ_{1122} and ρ_{1212}, and then measuring experimentally the ρ'_{1123}, we have a way of distinguishing the two environments.

To measure the ρ'_{1123}, we use $\mathbf{B} = (Bu, Bv, Bw)$ As in 2.1.2b the anisotropy axes form an angle φ with x_1' and x_2'. Thus

$$\Delta\rho = [\rho''_{11}(B)\cos^2\varphi + \rho''_{22}(B)\sin^2\varphi - \rho'_{11}]p^2 +$$
$$+ [\rho''_{11}(B)\sin^2\varphi + \rho''_{22}(B)\cos^2\varphi - \rho'_{22}]q^2 \qquad (54)$$
$$+ \sin2\varphi[\rho''_{11}(B) - \rho''_{22}(B)]pq.$$

In this case the following equations are valid

$$(\rho'_{1111}u^2 + \rho'_{1122}v^2 + \rho'_{1133}w^2 + 2\rho'_{1123}vw)B^2 =$$
$$\rho''_{11}(\mathbf{B})\cos^2\varphi + \rho''_{22}(\mathbf{B})\sin^2\varphi - \rho'_{11}, \qquad (55)$$

$$(\rho'_{1122}u^2 + \rho'_{1111}v^2 + \rho'_{1133}w^2 - 2\rho'_{1123}vw)B^2 =$$
$$\rho''_{11}(\mathbf{B})\sin^2\varphi + \rho''_{22}(\mathbf{B})\cos^2\varphi - \rho'_{22}, \qquad (56)$$

and

$$2[2\rho'_{1123}wu + (\rho'_{1111} - \rho'_{1122})uv]B^2 =$$
$$= \sin2\varphi[\rho''_{11}(\mathbf{B}) - \rho''_{22}(\mathbf{B})], \qquad (57)$$

where now the $\rho''_{ii}(B)$ are along the new axes of anisotropy.

Since we have already determined ρ'_{1111}, ρ'_{1122} and ρ'_{1133} eqs. (55), (56) and (57) identify three possible methods to measure ρ'_{1123}. The experimental procedure is similar to that presented in 2.1.2b. To simplify matters, we may choose all the angles 45° so that $u = 1/2$, $v = 1/2$, $w = 1/\sqrt{2}$.

3. ANISOTROPIC BEHAVIOUR DUE TO MECHANICAL TENSIONS

This section is referred to experimental results from Ge samples in (111) plane. Measurements were made on several samples of different sources. The behaviour of all of them was quite the same in respect to their polar distribution of $R(\mathbf{B})$.

If we choose as the system of reference the previous one described in 2.3, the zero-field resistivity will be anisotropic with axes in the directions $x_1 = [1\bar{1}0]$ and $x_2 = [11\bar{2}]$. This unexpected behaviour seems to be associated with mechanical tensions introduced probably by samples formation and has significant influence on the magnetoresistance

TABLE III.

Experimental aud calculated results with $B = 1.4\ T$ (SI units).

Direction of **B**	$(R_1/R_2)_{max}$	λ	Angle of skewness	ρ_{11}	ρ_{22}	$\rho'_{22}(\mathbf{B})$	$\rho'_{11}(\mathbf{B})$	$<\rho>$	ΔR_{12}
zero-field	1.0824	1.0278	—	4.454×10^{-2}	4.334×10^{-2}	—	—	4.394×10^{-2}	
0^0	1.0322	1.0112	0^0			4.861×10^{-2}	4.807×10^{-2}	4.834×10^{-2}	
15^0	1.00	1.00	indefinite			4.838	4.838	4.838	
30^0	1.0619	1.0211	90^0			4.890	4.789	4.840	
45^0	1.1479	1.0490	80^0			4.960	4.728	4.844	
60^0	1.2073	1.0674	60^0			5.002	4.686	4.844	
75^0	1.1160	1.0388	55^0			4.974	4.788	4.881	
90^0	1.0607	1.0206	90^0			4.975	4.875	4.925	
normal	1.1419	1.0471				$\rho_{11}(\mathbf{B})=5.202$	$\rho_{22}(\mathbf{B})=4.968$	5.085	—35.485

effect. In Fig. 9 we present the Van der Pauw "Resistance" R(0) and R(B) for several directions of the magnetic field **B** in the plane of the sample. The phenomenon of the magnetoresistance skewness is evident from this figure. So for $u = 1$ and $u = 0$ the angle of skewness is $\varphi = 0^0$ and $\varphi = 90^0$ respectively. This means that when the direction of **B** coincides with the x_1 axis, the anisotropy has principal axes which are the same as the zero-field resistance ones while when the direction of B coincides with the x_2 axis there is an exchange in the maximum and minimum directions. Since we don't know the "symmetry" of the zero-field resistivity we can't predict the angle of skewness for values of u between 1 and 0. But from Fig. 9 we may conclude that the skewness

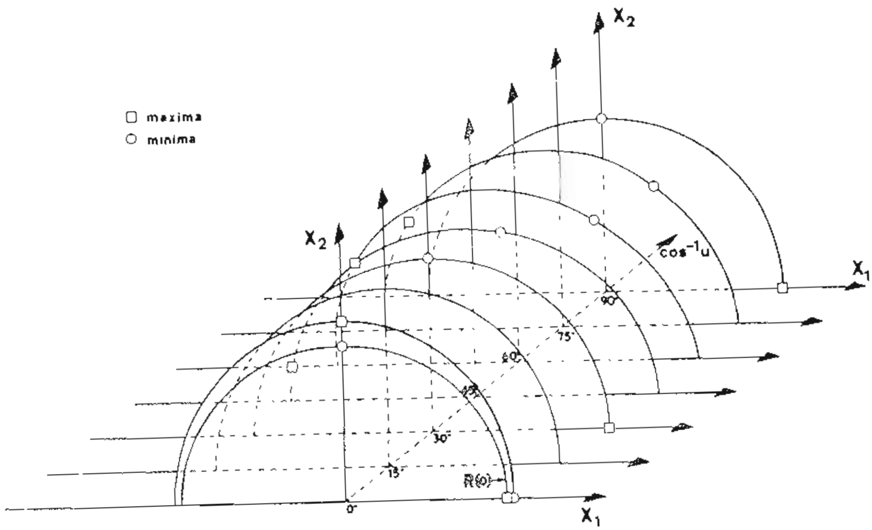

Fig. 9. The van der Pauw resistances R(0) and R(B) for B parallel to the (111) sample plane showing the anisotropy of the resistivity.

seems to be associated with the succession of similar axes every 60^0 and the exchange of axes of the form $[1\bar{1}0]$ and $[11\bar{2}]$ every 30^0 on the plane of the sample. With **B** perpendicular to the plane of the sample the magnetoresistance anisotropy has the same axes as the zero-field resistance. In table III the results of the measurements are presented. As we see with the magnetic field on the plane of the sample the values $\rho_{11}(\mathbf{B})$ and $\rho_{22}(\mathbf{B})$ are not constant and depend on the values of u, but the mean value of resistivity is almost constant[13].

Using these experimental values we can determine the following GVM coefficients [18].

$$R_H = \rho_{123} = -1.384 \times 10^{-2} m^3 C^{-1}$$
$$\rho_{1111} = 1.801 \times 10^{-3} m^3 C^{-1} T^{-1}$$
$$\rho_{2211} = 2.688$$
$$\rho_{1122} = 2.658$$
$$\rho_{2222} = 2.760$$
$$\rho_{1133} = 3.815$$
$$\rho_{2233} = 3.235$$

Final the number of carriers is given by the relation

$$n = \frac{1{,}18}{R_H e} \qquad (58)$$

so

$$n = 5.329 \times 10^{20}\, m^{-3} = 5.329 \times 10^{14}\, cm^{-3}$$

The existence of at least six different WFMR coefficients shows that the "symmetry" of anisotropy is very low and it is probable lower than the orthorombic one.

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ΠΕΡΙΛΗΨΗ

ΓΑΛΒΑΝΟΜΑΓΝΗΤΙΚΕΣ ΜΕΤΡΗΣΕΙΣ ΓΙΑ ΤΗ ΔΙΑΚΡΙΣΗ ΑΝΑΜΕΣΑ ΣΕ ΔΙΑΦΟΡΕΤΙΚΑ ΠΕΡΙΒΑΛΛΟΝΤΑ Ή ΓΙΑ ΤΗΝ ΕΞΑΚΡΙΒΩΣΗ ΑΝΙΣΟΤΡΟΠΗΣ ΣΥΜΠΕΡΙΦΟΡΑΣ ΑΝΕΞΑΡΤΗΤΗΣ ΑΠΟ ΤΗΝ ΚΡΥΣΤΑΛΛΙΚΗ ΣΥΜΜΕΤΡΙΑ

Ὑπὸ

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Στὴν ἐργασία αὐτὴ ἀφοῦ δίνεται μιὰ σύντομη περιγραφὴ τῶν παραδοσιακῶν γαλβανομαγνητικῶν μετρήσεων ἀσθενοῦς πεδίου τονίζεται ἡ σπουδαιότητα καὶ ἡ χρήση τῆς μεθόδου Wasscher γιὰ τὴν ἐξακρίβωση ἀνισότροπης ἠλεκτρικῆς συμπεριφορᾶς ποὺ δὲν ὀφείλεται στὴν κρυσταλλικὴ συμμετρία τοῦ ὑλικοῦ. Στὴ συνέχεια περιγράφεται πῶς μπορεῖ νὰ γίνει διάκριση κυβικῆς ἢ μὴ κυβικῆς συμμετρίας τοῦ περιβάλλοντος στὴν περίπτωση ἐπιταξιακῶν ὑμενίων ἢ προσανατολισμένων στρωμάτων κατὰ τὰ ἐπίπεδα (001), (110) καὶ (111). Στὸ τέλος δίνονται πειραματικὰ ἀποτελέσματα ἀπὸ μετρήσεις σὲ δεῖγμα μονοκρυσταλλικοῦ Ge παράλληλο στὸ ἐπίπεδο (111) τὸ ὁποῖο ἔπαθε μηχανικὴ παραμόρφωση. Παρατηρήθηκε ἀνισοτροπία ἠλεκτρικῆς ἀντίστασης μηδενικοῦ πεδίου μὲ κύριες διευθύνσεις τοὺς ἄξονες $[1\bar{1}0]$ καὶ $[11\bar{2}]$. Μὲ μεταβολὴ τῆς διεύθυνσης τοῦ μαγνητικοῦ πεδίου B παράλληλα στὸ ἐπίπεδο τοῦ δείγματος παρατηρήθηκε τὸ φαινόμενο τῆς ἑλίκωσης τῆς μαγνητοαντίστασης. Ἀνισότροπη μαγνητοαντίσταση παρατηρήθηκε καὶ μὲ τὸ πεδίο κάθετο στὸ ἐπίπεδο τοῦ δείγματος. Ἔτσι συνολικὰ προσδιορίστηκαν ἕξι διαφορετικοὶ συντελεστὲς μαγνητοαντίστασης ποὺ δείχνουν ὅτι ἡ «συμμετρία» τῆς ἀνισοτροπίας εἶναι πολὺ χαμηλότερη ἀπ' αὐτὴν τῆς κυβικῆς στὸ ἐπίπεδο (111).