Sci. Annals, Fac. & Mathem., Univ. Thessaloniki, **18**, 129 (1978)

APPLICATION OF THE NON-UNITARY MODEL OPERATOR APPROACH TO THE ^{16}O NUCLEUS

by
S. E. MASSEN

(Department of Theoretical Physics, University of Thessaloniki - Thessaloniki, Greece)

(Reeeived 14.10.78)

Abstract: *The non-unitary model operator approach which has been previously described, is applied to the ^{16}O nucleus. Approximate expressions are given for the ground state energy of this nucleus, by using both variational methods with the separation condition developed in our previous work. Simple hard and soft core potentials are employed in the computations and the results obtained with the two methods are discussed.*

1. Introduction

A non-unitary model operator approach to two-body correlations in finite nuclei has been described in references 9 and 10. Two approximate expressions for the ground state energy of closed shell nuclei have been derived in a general form and detailed investigations have been performed for the simplest case, namely that of the ^4He nucleus. The approximate expression for $\langle E\rangle$ in the two methods for this nucleus were given in terms of the matrix elements of the effective interaction M_{nlS} and the normalization integrals N_{nlS}.

The object of the present paper is to give the corresponding approximate expressions for the energy of the ^{16}O nucleus and to report the results of the computations based on them. The calculations in the case of ^{16}O are more complicated than in the case of ^4He, because there are now additional states in the expression for the energy besides the states (nlS) = (00S) and the Euler-Lagrange equations of the states (nlS) = (00S) and (10S) are coupled. These calculations are exhibited in sections 2 and 3. In section 4, the numerical values of the energy of this nucleus are given for various values of the oscillator parameter $b_l = (\hbar/M\omega)^{1/2}$, using both approximate expressions. In performing our computations the potentials of Kallio-Kolltveit (KK)([8]), Moszkowski - Scott (MS) [11], Ohmura - Morita - Yumada (OMY)[12], S1[1] and Harada - Tamagaki - Tanaka[7] (HTT) have been used. Finally, some details on the calculations of the two-body part of the energy expectation value: $(\Delta E)_2$ for ^{16}O are given in the appendix.

2. The expression for the ground state energy of ^{16}O in the first method

The general approximate expression for the ground state energy, $\langle E \rangle$ of the closed shells nuclei which was found with the non-unitary model operator approach and with the first method has been given in reference 9. This is the following

$$\langle E \rangle = \langle T_0 \rangle + (\Delta E)_2 + \ldots\ldots \tag{1}$$

where $\langle T_0 \rangle$ is the expectation value of the kinetic energy operator of the ground state in the independent-particle model, which is chosen to be the oscillator shell model and $(\Delta E)_2$ is given by

$$(\Delta E)_2 = \sum_{i<j}^{A} \left[\frac{\sum_{nls} \left[C^{ij}_{nls} M_{nls} + C^{ij}_{(n,n+1)ls} \langle \psi_{nls} | \psi_{n+1,ls} \rangle + C^{ij}_{(n,n-1)ls} \langle \psi_{nls} | \psi_{n-1,ls} \rangle \right]}{\sum_{nls} C^{ij}_{nls} \langle \psi_{nls} | \psi_{nls} \rangle} \right] \tag{2}$$

The expressions of the coefficients C^{ij}_{nls}, $C^{ij}_{(nn+1)ls}$ and the matrix element M_{nls} have been give in reference 9 (formulae 14, 25).

The variation of $\langle E \rangle$ with respect to the correlated relative trial wave function Ψ_{nls} by using also the separation condition has led to the Euler equation

$$-\frac{\hbar^2}{M}\frac{d^2\psi_{nls}}{dr^2} + \left[\frac{\hbar^2}{M}\frac{l(l+1)}{r^2} + \upsilon_{ls}(r) - \frac{E_{nl}}{2} - \varepsilon_{nls} \right] \psi_{nls} =$$

$$= -\frac{B_{n+1,ls}}{2}\psi_{n+1,ls} - \frac{B_{n-1,ls}}{2}\psi_{n-1,ls} \tag{3}$$

$$(c < r < d)$$

where the general expressious of the quantities $B_{n\pm1,ls}$ and ε_{nls} have been given in reference 9 (formulae 18, 19).

It is clear that the Euler equation for the correlated relative wave functions are generally coupled. This coupling, which does not exist in the case of the nucleus ^4He, where the quantum number n is only zero, exists in the case of the nucleus ^{16}O, where n takes the values 0 and 1.

In order to find the expression of $\langle E \rangle$, in the case of ^{16}O nucleus the expression of $(\Delta E)_2$ must be found. The expression of $\langle T_0 \rangle$ is well known:

$$\langle T_0 \rangle = \sum_{n_i l_i} \left(2n_i + l_i + \frac{3}{2} \right) \frac{\hbar\omega}{2} = 18\hbar\omega \tag{4}$$

For convenience we separate the sum $\sum_{i<j} [\quad]$ in expression (2), into three sums

$$(\Delta E)_2 = \sum_{i<j} [\quad] = \sum_{\alpha} [\quad] + \sum_{\beta} [\quad] + \sum_{\gamma} [\quad] \tag{5}$$

where we sum over pairs of nucleons with the following quantum numbers:

α) $n_i = 0,\ l_i = 0,\ n_j = 0,\ l_j = 0$

β) $n_i = 0,\ l_i = 0,\ n_j = 0,\ l_j = 1$

γ) $n_i = 0,\ l_i = 1,\ n_j = 0,\ l_j = 1$

After a long calculation, some details of which are given in the appendix, we arrive at the following expression for the term $(\Delta E)_2$:

$$(\Delta E)_2 = A_{000}M_{000} + A_{001}M_{001} + A_{010}M_{010} + A_{011}M_{011} + A_{020}M_{020} +$$
$$+ A_{021}M_{021} + A_{100}M_{100} + A_{101}M_{101} - \sqrt{\frac{3}{2}}\,\hbar\omega (A_{100} \langle \psi_{000} | \psi_{100} \rangle +$$
$$+ A_{101} \langle \psi_{001} | \psi_{101} \rangle) \tag{6}$$

The quantities A_{nlS}, which depend on the normalization integrals $\langle \psi_{nlS} | \psi_{nlS} \rangle$ of the various relative states, are given in the appendix.

Using the expressions of $\langle T_0 \rangle$ and $(\Delta E)_2$, the approximate energy expression of ^{16}O, if we include the centre of mass correction and the Coulomb energy[4], takes the following form

$$\langle E \rangle = 18\hbar\omega - \frac{3}{4}\hbar\omega + \frac{83}{2\sqrt{2\pi}} \frac{e^2}{b_1} + \sum_{S} [A_{00S}M_{00S} + A_{01S}M_{01S} +$$
$$+ A_{02S}M_{02S} + A_{10S}(M_{10S} - \sqrt{\frac{3}{2}}\,\hbar\omega \langle \psi_{00S} | \psi_{10S} \rangle)] \quad (S = 0 \text{ and } 1) \tag{7}$$

We may note that if the model operator is unitary[2], (and therefore the correlated wave functions ψ_{nlS} are orthonormal) the expression of $\langle E \rangle$ will become:

$$\langle E \rangle = 17.25\hbar\omega + \frac{83}{2\sqrt{2\pi}} \frac{e^2}{b_1} + 21\,(M_{000} + M_{001}) + 6M_{010} + 54M_{011} +$$
$$+ 7.5(M_{020} + M_{021}) + 1.5(M_{100} + M_{101}) \tag{8}$$

This is indeed the expression of $\langle E \rangle$ in the case of the unitary model operator approach(6).

In order to obtain the value of $\langle E \rangle$ the matrix elements M_{nlS}, $\langle \psi_{00S} \mid \psi_{10S} \rangle$ and the quantities A_{nlS} have to be computed. The M_{nlS} are computed from equation (25) of reference 1 and the quantities A_{nlS} from equations (A.9) to (A.13) of the appendix, after solving the Euler equations for the various states. It must be noted that in the case of ^{16}O the Euler equations for the states (nlS) = (00S) and (nlS) = (10S) are coumpled while the states (nlS) = (01S) and (nlS) = (02S) they are not coupled. The expressions of the quantities $\varepsilon_{nlS}, B_{n\pm1,lS}$ for the various states could be found from the general formulae (19) and (18) of reference 9, following a procedure similar to that for the expression of $(\Delta E)_3$. Such a procedure is however laborious and the expressions of these quantities were therefore obtained by applying the variational principle directly to the expression (7) for the energy of ^{16}O. In this way we arrived at the following expressions:

$$\varepsilon_{00S} = \frac{Q_{00S}}{A_{00S}} \quad , \quad B_{0+1,0S} = -\sqrt{\frac{3}{2}}\,\hbar\omega\,\frac{A_{10S}}{A_{00S}} \quad , \quad B_{0-1,0S} = 0 \tag{9}$$

$$\varepsilon_{10S} = \frac{Q_{10S}}{A_{10S}} \quad , \quad B_{1+1,0S} = 0 \quad , \quad B_{1-1,0S} = -\sqrt{\frac{3}{2}}\,\hbar\omega \tag{10}$$

$$\varepsilon_{01S} = \frac{Q_{01S}}{A_{01S}} \quad , \quad B_{0+1,1S} = B_{0-1,1S} = 0 \tag{11}$$

$$\varepsilon_{02S} = \frac{Q_{02S}}{A_{02S}} \quad , \quad B_{0+1,2S} = B_{0-1,2S} = 0 \tag{12}$$

The expressions of the numerators Q_{nlS} are given in the appendix.

3. The expression of the ground state energy of ^{16}O in the second method

The energy expression which was found with the second method of the non-unitary model operator approach of reference 9 is as follows:

$$\langle E \rangle = E_0 + (\widetilde{\Delta E})_2 + \ldots \tag{13}$$

where $$E_0 = 2\langle T_0\rangle \qquad (14)$$

and

$$(\widetilde{\Delta E})_2 = \sum_{i<j}^{A} \left[\frac{\sum_{nlS} [C^{ij}_{nlS}\widetilde{M}_{nlS} + \widetilde{C}^{ij}_{(n,n+1)lS} \langle\psi_{nlS}|\psi_{n+1,lS}\rangle + \widetilde{C}^{ij}_{(n,n-1)lS} \langle\psi_{nlS}|\psi_{n-1,lS}\rangle]}{\sum_{nlS} C^{ij}_{nlS} \langle\Psi_{nlS}|\Psi_{nlS}\rangle} \right]$$

$$- \sum_{i<j}^{A} \left[\frac{\sum_{nlS} G^{ij}_{nlS} \langle\psi_{nlS}|\psi_{nlS}\rangle}{\sum_{nlS} C^{ij}_{nlS} \langle\psi_{nlS}|\psi_{nlS}\rangle} \right] \qquad (15)$$

The expressions of the coefficients $\widetilde{C}^{ij}_{(n,n+1)ls}$ are similar to those of the first method(9,10). They differ only in that instead of the matrix element $\langle NL|\hat{t}_R|N \mp 1,L\rangle$ which appears in the coefficients $C^{ij}_{(n,n+1ls)}$ there now appears the matrix element $\langle NL \left| \frac{-\upsilon_R}{A-1} \right| N \mp 1,L\rangle = \frac{1}{A-1}$ $\langle NL|\hat{t}_R|N \mp 1,L\rangle$. The coefficients G^{ij}_{nls} are similar to C^{ij}_{nls}. They contain also the factor $\frac{E_{NL}}{2(A-1)}$. The matrix elements $\widetilde{M}_{nls}$ have been given in reference 9.

The Euler equation for the ψnlS in this method is:

$$-\frac{\hbar^2}{M}\frac{d^2\psi_{nlS}}{dr^2} + \left[\frac{\hbar^2}{M}\frac{l(l+1)}{r^2} + \frac{A-2}{A-1}\frac{\hbar^2}{M}\frac{r^2}{b^4} + \upsilon_{lS}(r) - E_{nl} - \widetilde{\varepsilon}_{nlS}\right]\psi_{nlS} =$$

$$= -\frac{\widetilde{B}_{n+1,lS}}{2}\psi_{n+1,lS} - \frac{\widetilde{B}_{n-1,lS}}{2}\psi_{n-1,lS} \quad (c<r<d) \qquad (16)$$

where $b = (2\hbar/M\omega)^{1/2}$ is the harmonic-oscillator parameter for the relative motion.

The expression of $(\widetilde{\Delta E})_2$ in the case of ^{16}O nucleus is found as follows:

The first sum of expression (15), which we call $(\widetilde{\Delta E})_{2a}$ is similar to $(\Delta E)_2$ of the first method. The expression of $(\widetilde{\Delta E})_{2a}$ can be found from the known expression of $(\Delta E)_2$ (expression (6)), if instead of M_{nls} and $\sqrt{\frac{3}{2}}\,\hbar\omega$ we put $\widetilde{M}_{nls}$ and $\frac{1}{15}\sqrt{\frac{3}{2}}\hbar\omega$. The expression which is found is the following:

$$(\widetilde{\Delta E})_{2a} = A_{000}\widetilde{M}_{000} + A_{001}\widetilde{M}_{001} + A_{010}\widetilde{M}_{010} + A_{011}\widetilde{M}_{011} + A_{020}\widetilde{M}_{020} + A_{021}\widetilde{M}_{021} +$$

$$+ A_{100}\widetilde{M}_{100} + A_{101}\widetilde{M}_{101} - \frac{1}{15}\sqrt{\frac{3}{2}}\,\hbar\omega(A_{100}\langle\psi_{000}|\psi_{100}\rangle + A_{101}\langle\psi_{001}|\psi_{001}\rangle) \qquad (17)$$

The second sum of (15), which we call $(\widetilde{\Delta E})_{2b}$ is found by following the same procedure as in the case of $(\Delta E)_2$ of the first method. This is:

$$(\widetilde{\Delta E})_{2b} = \frac{1}{30}\Big[(A_{000}N_{000} + A_{001}N_{001})\frac{7}{2}\hbar\omega + (A_{010}N_{010} + A_{011}N_{011})\frac{5}{2}\hbar\omega +$$

$$+ (A_{020}N_{020} + A_{021}N_{021})\frac{3}{2}\hbar\omega + (A_{100}N_{100} + A_{101}N_{101})\frac{3}{2}\hbar\omega\Big] - 2\hbar\omega \quad (18)$$

where $\qquad N_{nlS} = \langle\psi_{nlS}|\psi_{nlS}\rangle$

Finally, by using equations (14), (17), (18) as well as the expression for the correction of the center-of-mass motion and that of the Coulomb energy, the expression of ^{16}O in the second method becomes:

$$\langle E\rangle = 35{,}25\hbar\omega + \frac{83}{2\sqrt{2\pi}}\frac{e^2}{b_1^2} + 2\hbar\omega + \sum_S\Big[A_{00S}(\widetilde{M}_{00S} - \frac{7}{60}\hbar\omega N_{00S}) +$$

$$+ A_{01S}(\widetilde{M}_{01S} - \frac{5}{60}\hbar\omega N_{01S}) + A_{02S}(\widetilde{M}_{02S} - \frac{3}{60}\hbar\omega N_{02S}) + A_{10S}(\widetilde{M}_{10S} -$$

$$- \frac{3}{60}\hbar\omega N_{10S}) - \frac{1}{15}\sqrt{\frac{3}{2}}\hbar\omega A_{10S}\langle\psi_{00S}|\psi_{10a}\rangle\Big]$$

$$(S = 0 \text{ and } 1) \qquad (19)$$

The expressions of the quantities $\widetilde{\varepsilon}_{nlS}$, $\widetilde{B}_{n+1,lS}$ for the various states are

$$\widetilde{\varepsilon}_{00S} = \frac{\widetilde{Q}_{00S}}{A_{00S}} \qquad \widetilde{B}_{0+1,0S} = -\frac{1}{15}\sqrt{\frac{3}{2}}\,\hbar\omega\,\frac{A_{10S}}{A_{00S}} \qquad \widetilde{B}_{0-1,0S} = 0 \qquad (20)$$

$$\widetilde{\varepsilon}_{01S} = \frac{\widetilde{Q}_{01S}}{A_{01S}} \qquad \widetilde{B}_{0+1,0S} = \widetilde{B}_{0-1,1S} = 0 \qquad (21)$$

$$\widetilde{\varepsilon}_{02S} = \frac{\widetilde{Q}_{02S}}{A_{02S}} \qquad \widetilde{B}_{0+1,2S} = \widetilde{B}_{0-1,2S} = 0 \qquad (22)$$

$$\tilde{\varepsilon}_{10S} = \frac{\tilde{Q}_{10S}}{A_{10S}} \qquad \tilde{B}_{1+1,0S} = 0 \qquad \tilde{B}_{1-1,0S} = -\frac{1}{15}\sqrt{\frac{3}{2}}\hbar\omega \tag{23}$$

The expressions of the numerators $\tilde{Q}_{nlS}$ are of similar form to those in the previous case. They now contain the quantities $(\tilde{M}_{00S} - \frac{7}{60}\hbar\omega N_{00S})$, $(\tilde{M}_{01S} - \frac{5}{60}\hbar\omega N_{01S})$, $(\tilde{M}_{02S} - \frac{3}{60}\hbar\omega N_{02a})$, $(\tilde{M}_{10S} - \frac{3}{60}\hbar\omega N_{10S})$ instead of the matrix elements M_{00S}, M_{01S}, M_{10S} and the term $\frac{1}{15}\sqrt{\frac{3}{2}}\hbar\omega$ instead of $\sqrt{\frac{3}{2}}\hbar\omega$.

4. Results of numerical calculations

The procedure in computing the ground state energy of the ^{16}O nucleus is the following:

For a given potential and harmonic-oscillator parameter $b_1 = b/\sqrt{2}$, the Euler equations for the various states are solved numerically with arbitrary values of ε_{nlS} and $B_{0+1,0S}$, and the corresponding values of M_{nlS} and N_{nl}, are computed for various values of the separation distance,

The appropriate value of d in each case is the «variational Moszkowski and Scott separation distance», d_{MS} at which the wave function has also continuous derivative. In the case when more than one d_{MS} appear one may choose the smallest one. This choice might be physically interesting, since the short range of the correlations makes probable that the magnitude of the neglected higher terms in $\langle E \rangle$ is sufficiently small. The usual criterion for the fulfilment of this requirement is the smallness of the value of the corresponding healing integral

$$\eta_{nlS} = \int_0^\infty |\psi_{nlS} - \varphi_{nl}|^2 dr \tag{24}$$

The wave functions, which have been obtained in the manner, previously described, are used to caclulate new values for ε_{nlS} and $B_{0+1,0S}$ from the expressions (9) to (12) and the corresponding ones of the second method. This procedure is repeated until the values of each of the ε and B remain unchanged. These quantities are therefore determined in the present approach self-consistently.

As it is noted in chapter 2, the Euler equations for the correlated relative wave functions are generally coulped. These coupled equations were solved as follows:

The corresponding homogeneous differential equations were solved and their solutions were taken as the corresponding non-homogeneous parts of the equations. Having known the non-homogeneous parts of the equations, these were solved and their solutions were taken as the new non-homogenous parts and so on. Self-consistency was achieved after three repetitions.

In the computations we used for the nucleon-nucleon interaction the Serber-type potentials which were mentioned in the introduction. The potentials KK, OMY and MS are hard core, while the S1 and HTT are soft core potentials. The above potentials can be written in the form:

$$\upsilon(r) = \frac{1}{2}(1+\hat{P})\upsilon_t(r) + \frac{1}{2}(1-\hat{P})\upsilon_s(r) \tag{25}$$

Where $\hat{P}$ is the spin exchange operator and $\upsilon_t(r)$ and $\upsilon_s(r)$ the nucleon-nucleon interaction in the triplet and singlet state, respectively.

The form of $\upsilon_t(r)$ and $\upsilon_s(r)$ for the potentials KK, OMY and MS is the following:

$$\upsilon_{t,s}(r) = \begin{cases} \infty & \text{for } 0<r<c \\ -V_{t,s}\exp\{-\lambda_{t,s}(r-c)\} & \text{for } c<r<\infty \end{cases} \tag{26}$$

The parameters V_t, V_s, λ_t, λ_s and c are given in table 1.

TABLE 1

Parameters of the potentials KK, OMY, MS.

Potential	c(fm)	V_t(MeV)	V_s(MeV)	$\lambda_t(fm^{-1})$	$\lambda_s(fm^{-1})$
KK	0.4	475.0	330.8	2.5214	2.4021
OMY	0.4	475.044	235.414	2.5214	2.0344
MS	0.4	260.0	260.0	2.083	2.083

The $\upsilon_t(r)$ and $\upsilon_s(r)$ for the potential S1 have the form

$$\upsilon_{t,s}(r) = \sum_{i=1}^{3} V_{it,s}\exp\{-\alpha_{it,s}r^2\} \tag{27}$$

while for the potential HTT are

$$\upsilon_{t,s}(r) = \sum_{i=1}^{3} V_{it,s} \exp\{-(r/\alpha_{it,s})^2\} \tag{28}$$

The parameters of the potentials S1 and HTT are given in table 2.

TABLE 2

Parameters of the potentials S1, HTT.

Potential	state	V_1(MeV)	V_2(MeV)	V_3(MeV)	α_1	α_2	α_3
S1	triplet	1000	—143.4	—43.0	5.4fm^{-2}	0.82fm^{-2}	0.60fm^{-2}
S1	singlet	880	— 67.1	—21.0	5.2fm^{-2}	0.62fm^{-2}	0.38fm^{-2}
HTT	triplet	4000	—279.0	— 7.2	0.385fm	0.942fm	1.876fm
HTT	singlet	4000	—279.0	— 7.2	0.385fm	0.942fm	1.876fm

The computed values of <E> for the ^{16}O nucleus, using the above potentials for some values of the harmonic-oscillator parameter b_1 and the first method, are given in table 3 (see also figure 1). The various contributions to the ground state energy are aslo given in this table, in which T_{CM} is the correction due to the center of mass motion and E_C the Coulomb energy, estimated from the oscillator wave functions.

The results of our computations for various values of b_1 show that for small values of this parameter no acceptable d_{MS} appear for the state (100). These values of b_1 are noted by an asterisk above the value of <E> in table 3 and by a dotted curve in figure 1. It is seen from table 3 and figure 1 that there is no minimum in the saturation curves for all the potentials. In order to estimate the value of <E> we may use the value, $b_1 = 1.764$ fm (or $\hbar\omega = 13.33$ MeV), which is determined from the analysis of the experiments of the elastic scattering of electrons by ^{16}O[11]. For this value of b_1 and for the potentials KK, OMY and S1 the values of <E> which are computed are close enough to the experimental value (—127.52) MeV. The computed values of <E> for the potentials MS and HTT and for the same value of b_1 are bigger than the experimental value.

The results of our computations for some values of b_1 using the second method and the potentials KK, OMY and S1 are shown in table 4 and in figure 2. The saturation curves for these potentials have minimum corresponding to a negative energy. It should be noted that for the other two potentials the energy is positive for all the computed values of b_1.

TABLE 3

The values of the terms contributing to <E> for various values of b_1 for the potentials KK, OMY, MS, S1, HTT and the first method (lengths in fm, energies in MeV).

				KK		OMY		MS		S1		HTT	
b_1	$\hbar\omega$	$\langle T_O\rangle - T_{CM}$	E_C	$(\Delta E)_2$	$\langle E\rangle$	$(\Delta E)_2$	$\langle E\rangle$	$(\Delta E)_2$	$\langle E\rangle$	$(\Delta E)_2$	$\langle E\rangle$	$(\Delta E)_2$	$\langle E\rangle$
1.4	21.155	364.915	17.122	—551.606	—169.570*	—551.349	—169.312*	—480.566	—98.529*	—543.369	—161.332*	—435.690	—53.654
1.5	18.435	318.009	15.984	—485.552	—151.558*	—483.062	—149.069*	—425.957	—91.963	—480.013	—146.020*	—383.270	—49.276
1.6	16.194	279.353	14.981	—426.989	—132.656	—423.895	—129.561*	—376.197	—81.863	—423.856	—129.523*	—338.021	—43.687
1.7	14.350	247.544	14.102	—376.823	—115.176	—373.522	—111.876	—333.993	—72.347	—375.147	—113.501	—298.914	—37.268
1.764	13.333	230.001	13.593	—348.375	—104.781	—345.184	—101.590	—309.884	—66.290	—347.383	—103.789	—276.651	—33.057
1.8	12.794	220.702	13.316	—333.099	—99.082	—330.003	—95.986	—296.935	—62.918	—332.445	—98.428	—264.679	—30.662
1.9	11.487	198.146	12.617	—295.510	—84.746	—292.779	—82.016	—264.941	—54.177	—295.596	—84.833	—235.171	—24.407
2.0	10.370	178.881	11.988	—262.921	—72.052	—260.601	—69.733	—236.980	—46.112	—263.524	—72.656	—209.510	—18.642
2.5	6.633	114.419	9.588	—154.024	—30.017	—152.163	—28.157	—140.442	—16.435	—155.111	—31.104	—122.487	1.520

TABLE 4

The values of the terms contributing to <E> for various values of b_1 for the potentials KK, OMY, S1 and the second method, (lengths in fm, energies in MeV).

				KK		OMY		S1	
b_1	$\hbar\omega$	$F_O - T_{CM}$	E_C	$(\Delta E)_2$	<E>	$(\Delta E)_2$	<E>	$(\Delta E)_2$	<E>
1.4	21.155	745.696	17.122	—770.988	—8.170	—802.410	—39.593	—768.092	—5.275
1.5	18.435	649.844	15.984	—679.495	—13.665	—705.721	—39.891	—678.093	—12.263
1.6	16.194	570.839	14.981	—599.643	—13.811	—622.483	—36.651	—599.461	—13.630
1.7	14.350	505.852	14.102	—531.260	—11.306	—551.443	—31.489	—531.872	—11.918
1.764	13.333	470.001	13.593	—492.566	—8.971	—511.386	—27.791	—493.547	—9.952
1.8	12.794	450.999	13.316	—471.796	—7.482	—490.556	—26.242	—472.951	—8.637
1.9	11.487	404.906	12.617	—420.760	—3.235	—437.255	—19.730	—422.259	—4.734
2.0	10.370	365.539	11.988	—376.527	1.000	—391.620	—14.094	—378.229	—0.702
2.5	6.633	233.813	9.588	—226.547	16.853	—237.176	6.224	—228.258	15.142

The minimum values of <E> for the potentials KK and S1 correspond to $b_1 \simeq 1.6$ fm ($\hbar\omega \simeq 16.194$ MeV) while for the potential OMY to $b_1 \simeq 1.5$ fm ($\hbar\omega \simeq 18.435$ MeV). The corresponding balues of <E>, however, are too far from the experimental values of the ground state energy.

TABLE 5

The values of the healing integral in the s states for various values of b_1 for the potential KK and the first method.

b_1	η_{000}	η_{001}	η_{100}	η_{101}
1.4	0.0127	0.0113	0.0222	0.0173
1.5	0.0103	0.0092	0.0167	0.0139
1.6	0.0084	0.0075	0.0133	0.0114
1.7	0.0070	0.0063	0.0109	0.0095
1.764	0.0062	0 0056	0 0096	0 0085
1.8	0.0058	0.0053	0.0090	0.0079
1.9	0.0050	0.0045	0.0076	0.0067
2.0	0.0042	0.0038	0.0065	0.0058
2.5	0.0021	0.0020	0.0032	0.0029

We may finally point out that the observed descrepancies should be mostly attributed to the omission of the higher terms in the cluster expansion and to the simplicity of the potentials. This is indicated also by the fact that the values of the healing integrals, η_{nlS}, become large for small b_1. This behaviour of the healing integrals is shown in table 5. In this table, we tabulate the values of η_{000}, η_{001}, η_{100}, η_{101} which were found for the potential KK for some values of b_1 and using the first method. The behaviour of the healing integrals for the other potentials and for the two methods are similar.

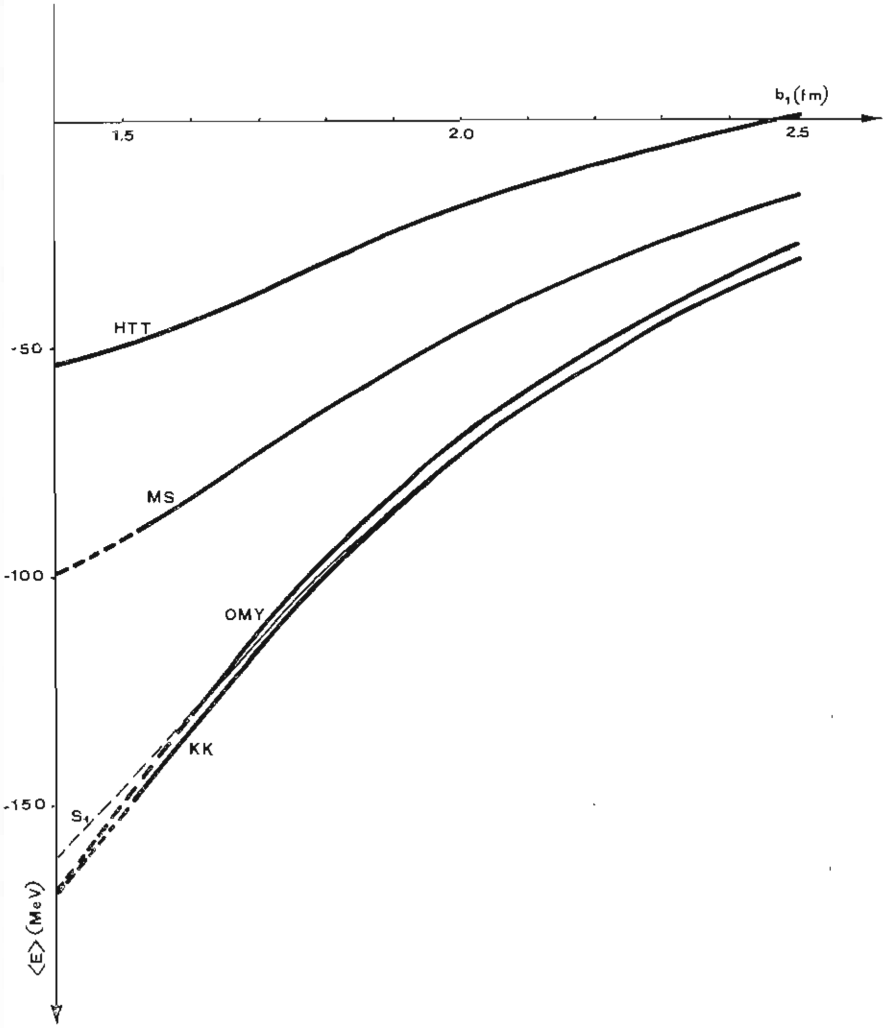

Fig. 1. The saturation curves for ^{16}O nucleus obtained with the potentials KK, OMY, MS, S1 and HTT and the first method.

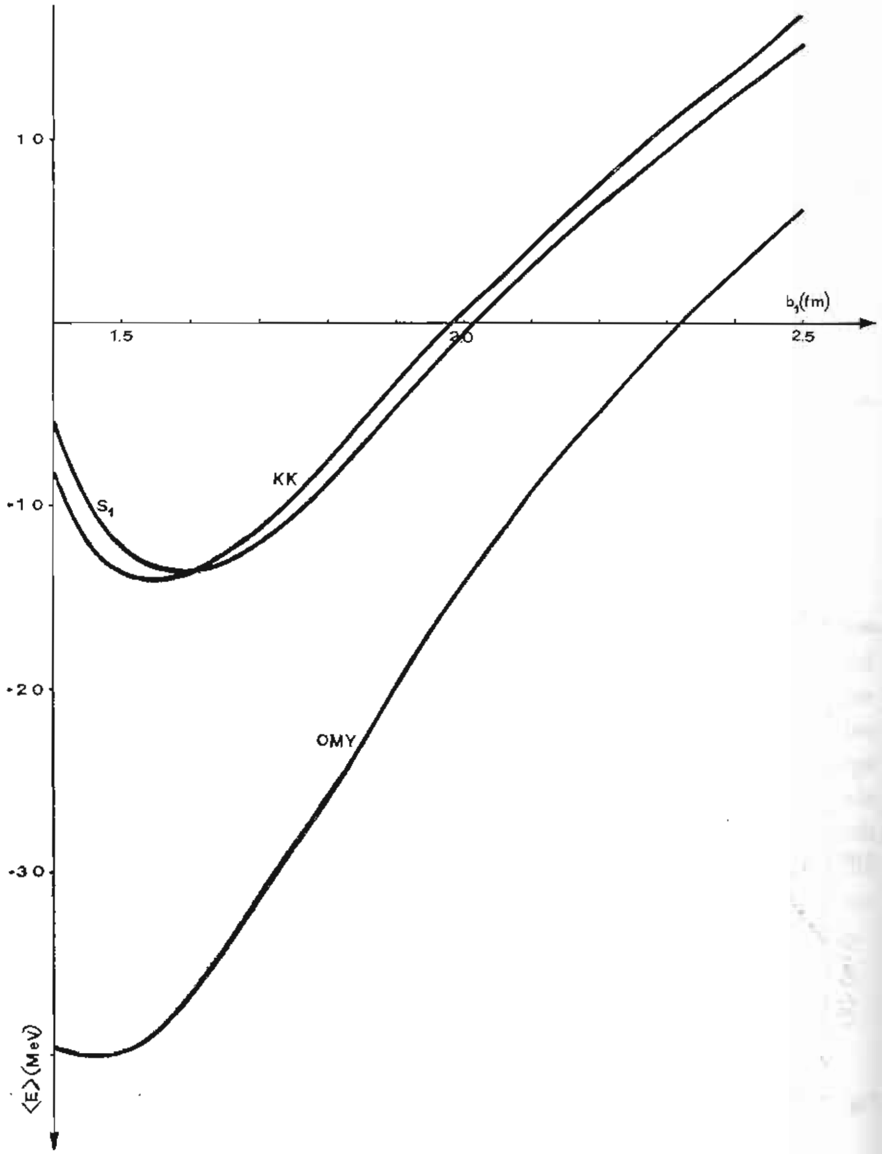

Fig. 2. The saturation curves for ^{16}O nucleus obtained with the potentials KK, OMY and S1 and the second method.

APPENDIX

Details on the calculation of the term $(\Delta E)_2$

In section 2 the term $(\Delta E)_2$ was separated in three sums (expression (5)).

In the first sum $\sum\limits_{\alpha} [\quad]$, we sum over the same pairs with the sum $\sum\limits_{i<j}^{4} [\quad]$ of the ^{4}He nucleus. Therefore the $\sum\limits_{\alpha} [\quad]$ is equal to the term $(\Delta E)_2$ for ^{4}He which has been given in references 1 and 2. This is:

$$\sum_{\alpha} [\quad] = \left[\frac{2}{N_{000}} + \frac{2}{N_{000}+N_{001}}\right] M_{000} + \left[\frac{2}{N_{001}} + \frac{2}{N_{000}+N_{001}}\right] M_{001} \quad (A.1)$$

In the second sum $\sum\limits_{\beta} [\quad]$, we sum over the set of pairs, which are characterized by the quantum numbers $n_i = l_i = m_i = 0$ and $n_j = 0$, $l_j = 1$, $m_i = 0 \pm 1$. The possible states of the relative motion and the motion of the center of mass can be found from the known relations

$$2n_i + l_i + 2n_j + l_j = 2n + l + 2N + L \quad (A.2a)$$

$$|l_i - l_j| \leq \lambda \leq l_i + l_j, \qquad |l - L| \leq \lambda \leq l + L \quad (A.2b)$$

$$(-1)^{l_i + l_j} = (-1)^{l+L} \quad (A.2c)$$

$$m_i + m_j = m + M = \mu \quad (A.2d)$$

Using these relations we see that the states of the relative motion and the motion of the center of mass are:

i) $n = 0,\ l = 0,\ m = 0 \quad N = 0,\ L = 1,\ M = 0, \pm 1 \quad (\lambda = 1,\ \mu = 0, \pm 1)$

ii) $n=0,\ l=1,\ m=0, \pm 1 \quad N = 0,\ L = 0,\ M = 0 \quad (\lambda = 1,\ \mu = 0, \pm 1)$

Since the quantum numbers n and N are equal to zero, the coefficients $C^{ij}_{(n,n+1)ls}$, of the non-diagonal term of the sum $\sum\limits_{\beta}[\quad]$, are zero. Therefore this sum is written as follows:

$$\sum_{\beta} [\quad] = \sum_{i<j} \left[\frac{\sum\limits_{0lS} [C^{ij}_{0ls} M_{0ls}]}{\sum\limits_{0lS} [C^{ij}_{0ls} N_{0ls}]}\right] \quad (A.3)$$

The numerator in this expression, taking into account the known expression of the coefficients C_{nls}^{ij} (9,10), can be writtenas follows:

$$\sum_{01S} C_{01s}^{ij} M_{01S} = \sum_{S}(C_{00s}^{ij} M_{00S} + C_{01s}^{ij} M_{01s}) =$$

$$= \sum_{S}\left[\frac{1}{2}\delta_{M_S 0}\delta_{S0} + (\delta_{M_S 1} + \frac{1}{2}\delta_{M_S 0} + \delta_{M_S -1})\delta_{S1}\right]\cdot\{<00,01:1 \mid 01,00:1>^2\cdot$$

$$\cdot <001m_j \mid 1\mu>^2[1 + (-1)^{2+S}\delta_{\tau_i\tau_j}]M_{00S} + <00,01:1 \mid 00,01:1>^2\cdot$$

$$\cdot <001m_j \mid 1\mu>^2 [1 + (-1)^{1+S}\delta_{\tau_i\tau_j}]M_{01s} \qquad (A.4)$$

Using the known values of the Clebsch-Gordan coefficient and taking the values of the Brody-Moshinsky brackets from tables(3), the expression (A.4) becomes:

$$\sum_{01S} C_{01s}^{ij} M_{01s} = \frac{1}{4}\delta_{MS0}[(1 + \delta_{\tau_i\tau_j})M_{000} + (1-\delta_{\tau_i\tau_j})M_{010}] +$$

$$+ \frac{1}{2}(\delta_{MS1} + \frac{1}{2}\delta_{MS0} + \delta_{MS-1})[(1-\delta_{\tau_i\tau_j})M_{001} + (1+\delta_{\tau_i\tau_j})M_{011}] \qquad (A.5)$$

The expression of the denominator of the equation (A.3) is similar to (A.5). It differs only in that instead of the M_{nlS} appears now the N_{nlS}. Substituting the expressions of the numerator and the denominator into (A.3) we get the following expression for $\sum_{\beta}[\quad]$:

$$\sum_{\beta}[\quad] = 12\left[\frac{M_{011}}{N_{011}} + \frac{M_{000} + M_{011}}{N_{000} + N_{011}} + \frac{M_{001} + M_{011}}{N_{001} + N_{011}} + \right.$$

$$\left. + \frac{M_{000} + M_{010} + M_{001} + M_{011}}{N_{000} + N_{010} + N_{001} + N_{011}}\right] \qquad (A.6)$$

In the third sum $\sum_{\gamma}[\quad]$, we sum over the set of pairs which are characterized by the quantum numbers $n_i = 0$, $l_i = 1$, $m_i = 0, \pm 1$, $n_j = 0$, $l_j = 1$, $m_j = 0, \pm 1$.

From the relations (A.2) we can get the possible states of the relative motion and the motion of the center of mass. These states are the following:

i) $n = 0,\ l = 0,\ m = 0 \qquad N = 1,\ L = 0,\ M = 0$

$(\lambda = 0,\ \mu = 0)$

ii) $n = 1,\ l = 0,\ m = 0$ $\qquad$ $N = 0,\ L = 0, M = 0$

$(\lambda = 0,\ \mu = 0)$

iii) $n = 0,\ l = 1,\ m = 0, \pm 1$ $\qquad$ $N = 0,\ L = 1,\ M = 0 \pm 1$

$(\lambda = 0,\ \mu = 0)$

iv) $n = 0,\ l = 1,\ m = 0, \pm 1$ $\qquad$ $N = 0,\ L = 1,\ M = 0, \pm 1$

$(\lambda = 1,\ \mu = 0, \pm 1)$

v) $n = 0,\ l = 0,\ m = 0$ $\qquad$ $N = 0,\ L = 2,\ M = 0, \pm 1, \pm 2$

$(\lambda = 2,\ \mu = 0, \pm 1, \pm 2)$

vi) $n = 0,\ l = 2,\ m = 0, \pm 1, \pm 2$ $\qquad$ $N = 0,\ L = 0,\ M = 0$

$(\lambda = 2,\ \mu = 0, \pm 1, \pm 2)$

vii) $n = 0,\ l = 2,\ m = 0, \pm 1$ $\qquad$ $N = 0,\ L = 1,\ M = 0, \pm 1$

$(\lambda = 2,\ \mu = 0, \pm 1, \pm 2)$

It is seen that the quantum numbers n and N are not always zero. The coefficients of the non-diagonal terms are not generally zero in this case and the sum $\sum_{\gamma} [\quad]$ will contain now and non-diagonal matrix elements. Following a procedure similar to that for the second sum the following expression is obtained:

$$\sum_{\gamma} [\quad] = \sum_{s} \Bigg[4 \frac{M_{00s} + M_{02s}}{N_{00s} + N_{02s}} + 8 \frac{M_{00s} + M_{02s} + 2M_{011}}{N_{00s} + N_{02s} + 2N_{011}} +$$

$$+ 4 \frac{3M_{00s} + M_{02s} + 2M_{10s} + 6M_{011} - 2\sqrt{\frac{3}{2}}\hbar\omega \langle\psi_{00s} | \psi_{10s}\rangle}{3N_{00s} + N_{02s} + 2N_{10s} + 6N_{011}} +$$

$$+ 2 \frac{3M_{00s} + 2M_{02s} + M_{10s} - \sqrt{\frac{3}{2}}\hbar\omega \langle\psi_{00s} | \psi_{10s}\rangle}{3N_{00s} + 2N_{02s} + N_{10s}} \Bigg] + 12 \frac{M_{011}}{N_{011}} +$$

$$+ 4 \frac{\sum_{s}(M_{00s} + M_{02s})}{\sum_{s}(N_{00s} + N_{02s})} + 8 \frac{\sum_{s}(M_{00s} + M_{02s} + 2M_{01s})}{\sum_{s}(N_{00s} + N_{02s} + 2N_{01s})} +$$

$$+ 4 \frac{\sum_{s}(3M_{00s} + M_{02s} + 2M_{10s} + 6M_{01s} - 2\sqrt{\frac{3}{2}}\hbar\omega \langle\psi_{00s} | \psi_{10s}\rangle)}{\sum_{s}(3N_{00s} + N_{02s} + 2N_{10s} + 6N_{01s})} +$$

$$+\,2\;\frac{\sum\limits_{S}\left(3M_{00\,s}+2M_{02\,s}+M_{10\,s}-\sqrt{\frac{3}{2}}\hbar\omega<\psi_{00\,s}|\psi_{10\,s}>\right)}{\sum\limits_{S}\left(3N_{00\,s}+2N_{02\,s}+2N_{10\,s}\right)}\quad (S=0 \text{ and } 1) \tag{A.7}$$

If we substitute (A.1), (A.6) and (A.7) into (5) the term $(\Delta E)_2$ becomes:

$$(\Delta E)_2=\sum_{S}\Big[A_{00\,s}M_{00\,s}+A_{01\,s}M_{01\,s}+$$

$$+\,A_{02\,s}M_{02s}+A_{10s}(M_{10\,s}-\sqrt{\frac{3}{2}}\hbar\omega<\psi_{00s}\,|\,\psi_{10\,s}>)$$

$$(S=0 \text{ and } 1) \tag{A.8}$$

where the quantities $A_{nl\,s}$, which depend on the normalization integrals $N_{nl\,s}$ are given by the following expressions:

$$A_{00\,s}=\frac{2}{N_{00\,s}}+\frac{2}{\sum\limits_{k}N_{00\,k}}+\frac{12}{N_{00\,s}+N_{011}}+\frac{12}{\sum\limits_{k}(N_{00\,k}+N_{01\,k})}+$$

$$+\frac{4}{N_{00\,s}+N_{02\,s}}+\frac{4}{\sum\limits_{k}(N_{00\,k}+N_{02\,k})}+\frac{8}{N_{00\,s}+N_{02\,s}+2N_{011}}+$$

$$+\frac{8}{\sum\limits_{k}(N_{00\,k}+N_{02\,k}+2N_{01\,k})}+\frac{12}{3N_{00\,s}+N_{02\,s}+2N_{10\,s}+6N_{011}}+$$

$$+\frac{12}{\sum\limits_{k}(3N_{00\,k}+N_{02\,k}+2N_{10\,k}+6N_{01\,k})}+\frac{6}{3N_{00\,s}+2N_{02\,s}+N_{10\,s}}+$$

$$+\frac{6}{\sum\limits_{k}(3N_{00\,k}+2N_{02\,k}+N_{10\,k})} \tag{A.9}$$

$$A_{010}=\frac{12}{\sum\limits_{k}(N_{00\,k}+N_{01\,k})}+\frac{16}{\sum\limits_{k}(N_{00\,k}+N_{02\,k}+2N_{01\,k})}+$$

$$+\frac{24}{\sum\limits_{k}(3N_{00\,k}+N_{02\,k}+2N_{10\,k}+6N_{01\,k})} \tag{A.10}$$

$$A_{011}=A_{010}+\frac{24}{N_{011}}+\sum_{k}\Big[\frac{12}{N_{00\,k}+N_{011}}+\frac{16}{N_{00\,k}+N_{02\,k}+2N_{011}}+$$

$$+ \frac{24}{3N_{00k} + N_{02k} + 2N_{10k} + 6N_{011}} \Bigg] \qquad (A.11)$$

$$A_{10\,s} = \frac{8}{3N_{00\,s} + N_{02\,s} + 2N_{10\,s} + 6N_{011}} + \frac{8}{\sum\limits_k (3N_{00\,k} + N_{02k} + 2N_{10\,k} + 6N_{01k})} +$$

$$+ \frac{2}{3N_{00\,s} + 2N_{02\,s} + N_{10\,s}} + \frac{2}{\sum\limits_k (3N_{00k} + 2N_{02k} + N_{10k})} \qquad (A.12)$$

$$A_{02\,s} = \frac{A_{10\,s}}{2} + \frac{4}{N_{00\,s} + N_{02\,s}} + \frac{4}{\sum\limits_k (N_{00k} + N_{02k})} + \frac{8}{N_{00\,s} + N_{02\,s} + 2N_{011}} +$$

$$+ \frac{8}{\sum\limits_k (N_{00k} + N_{02k} + 2N_{01k})} + \frac{3}{3N_{00\,s} + 2N_{02\,s} + XN_{10\,s}} +$$

$$+ \frac{3}{\sum\limits_k (3N_{00k} + 2N_{02k} + N_{10k})} \qquad (A.13)$$

In the above expressions the index k in the sums takes the values 0 and 1.

Finally the quantities $Q_{nl\,s}$ which are the numerators of $\varepsilon_{nl\,s}$ are given by the following expressions:

$$Q_{00\,s} = 2 \left[\frac{M_{00\,s}}{(N_{00\,s})^2} + \frac{\sum\limits_k M_{00\,k}}{(\sum\limits_k N_{00k})^2} + 6\,\frac{M_{00\,s} + M_{011}}{(N_{00\,s} + N_{011})^2} + \right.$$

$$+\ 6\,\frac{\sum\limits_k (M_{00k} + M_{01k})}{\left\{\sum\limits_k (N_{00k} + N_{01k}\right\}^2} + 2\,\frac{M_{00\,s} + M_{02\,s}}{(N_{00\,s} + N_{02\,s})^2} + 2\,\frac{\sum\limits_k (M_{00\,k} + M_{02k})}{\left\{\sum\limits_k (N_{00k} + N_{02k}\right\}^2} +$$

$$+\ 4\,\frac{M_{00\,s} + M_{02\,s} + 2M_{011}}{(N_{00\,s} + N_{02\,s} + 2N_{011})^2} + 4\,\frac{\sum\limits_k (M_{00k} + M_{02k} + 2M_{01k})}{\left\{\sum\limits_k (N_{00k} + N_{02k} + 2N_{01k}\right\}^2} +$$

$$+\ 6\,\frac{3M_{00\,s} + M_{02\,s} + 2M_{10\,s} + 6M_{011} - 2\sqrt{\frac{3}{2}}\,\hbar\omega \langle \psi_{00\,s} \mid \psi_{10\,s} \rangle}{(3N_{00\,s} + N_{02\,s} + 2N_{10\,s} + 6N_{011})^2} +$$

$$+ 6 \frac{\sum_k (3M_{00k} + M_{02k} + 2M_{10k} + 6M_{01k} - 2\sqrt{\frac{3}{2}}\, \hbar\omega \langle \psi_{00k} | \psi_{10k} \rangle)}{\{\sum_k (3N_{00k} + N_{02k} + 2N_{10k} + 6N_{01k})\}^2} +$$

$$+ 3 \frac{3M_{00s} + 2M_{02s} + M_{10s} - \sqrt{\frac{3}{2}}\, \hbar\omega \langle \psi_{00s} | \psi_{10s} \rangle}{(3N_{00s} + 2N_{02s} + N_{10s})^2} +$$

$$+ 3 \frac{\sum_k (3M_{00k} + 2M_{02k} + M_{10k} - \sqrt{\frac{3}{2}}\, \hbar\omega \langle \psi_{00k} | \psi_{10k} \rangle)}{\{\sum_k (3N_{00k} + 2N_{02k} + N_{10k})\}^2} \Bigg] \quad (A.14)$$

$$Q_{010} = 12 \frac{\sum_k (M_{00k} + M_{01k})}{\{\sum_k (N_{00k} + N_{01k})\}^2} + 16 \frac{\sum_k (M_{00k} + M_{02k} + 2M_{01k})}{\{\sum_k (N_{00k} + N_{02k} + 2N_{01k})\}^2} +$$

$$+ 24 \frac{\sum_k (3M_{00k} + M_{02k} + 2M_{10k} + 6M_{01k} - 2\sqrt{\frac{3}{2}}\, \hbar\omega \langle \psi_{00k} | \psi_{10k} \rangle)}{\{\sum_k (3N_{00k} + N_{02k} + 2N_{10k} + 6N_{01k})\}^2} \quad (A.15)$$

$$Q_{011} = Q_{010} + 24 \frac{M_{011}}{(N_{011})^2} + \sum_k \Bigg[12 \frac{M_{00k} + M_{011}}{(N_{00k} + N_{011})^2} + 16 \frac{M_{00k} + M_{02k} + 2M_{011}}{(N_{00k} + N_{02k} + 2N_{011})^2} +$$

$$+ 24 \frac{3M_{00k} + M_{02k} + 2M_{10k} + 6M_{011} - 2\sqrt{\frac{3}{2}}\, \hbar\omega \langle \psi_{00k} | \psi_{10k} \rangle}{(3N_{00k} + N_{02k} + 2N_{10k} + 6N_{011})^2} \Bigg] \quad (A.16)$$

$$Q_{10s} = 8 \frac{3M_{00s} + M_{02s} + 2M_{10s} + 6M_{011} - 2\sqrt{\frac{3}{2}}\, \hbar\omega \langle \psi_{00s} | \psi_{10s} \rangle}{(3N_{00s} + N_{02s} + 2N_{10s} + 6N_{011})^2} +$$

$$+ 8 \frac{\sum_k (3M_{00k} + M_{02k} + 2M_{10k} + 6M_{01k} - 2\sqrt{\frac{3}{2}}\, \hbar\omega \langle \psi_{00k} | \psi_{10k} \rangle)}{\{\sum_k (3N_{00k} + N_{02k} + 2N_{10k} + 6N_{01k})\}^2} +$$

$$+ 2 \frac{3M_{00s} + 2M_{02s} + M_{10s} - \sqrt{\frac{3}{2}}\, \hbar\omega \langle \psi_{00s} | \psi_{10s} \rangle}{(3N_{00s} + 2N_{02s} + N_{10s})^2} +$$

$$+ 2 \frac{\sum_k \left(3M_{00k} + 2M_{02k} + M_{10k} - \sqrt{\frac{3}{2}}\, \hbar\omega < \psi_{00k} | \psi_{10k} > \right)}{\{\sum_k (3N_{00k} + 2N_{02k} + N_{10k})\}^2} \qquad (A.17)$$

$$Q_{02s} = \frac{Q_{10s}}{2} + 4 \frac{M_{00s} + M_{02s}}{(N_{00s} + N_{02s})^2} + 4 \frac{\sum_k (M_{00k} + M_{02k})}{\{\sum_k (N_{00k} + N_{02k})\}^2} +$$

$$+ 8 \frac{M_{00s} + M_{02s} + 2M_{011}}{(N_{00s} + N_{02s} + 2N_{011})^2} + 8 \frac{\sum_k (M_{00k} + M_{02k} + 2M_{01k})}{\{\sum_k (N_{00k} + N_{02k} + 2N_{01k})\}^2} +$$

$$+ 3 \frac{3M_{00s} + 2M_{02s} + M_{10s} - \sqrt{\frac{3}{2}}\, \hbar\omega < \psi_{00s} | \psi_{10s} >}{(3N_{00s} + 2N_{02s} + N_{10s})^2} +$$

$$+ 3 \frac{\sum_k \left(3M_{00k} + 2M_{02k} + M_{10k} - \sqrt{\frac{3}{2}}\, \hbar\omega < \psi_{00k} | \psi_{10k} > \right)}{\{\sum_k (3N_{00k} + 2N_{02k} + N_{10k})\}^2} \qquad (A.18)$$

As in the previous expressions, the index k in the sums takes the values 0 and 1.

* * *

The compulations were performed in the computer of the University of Thessaloniki. The author would like to thank the computing staff for their co-operation. He would also like to thank Prof. M. E. Grypeos for useful suggestions and discussions.

REFERENCES

1. I. R. Afnan, Y. C. Tang (1968): Phys. Rev., 175, 4, 1337.
2. D. M. Brink, M. E. Grypeos (1967): Nucl. Phys., A97, 1.
3. T. A. . Brody, M. Moshinsky (1960): Tables of Transformation Brackets. Monografias del Instituto de Fisica.
4. J. Dabrowski (1958): Proc. Phys. Soc., 71, 658.
5. L. R. B. Elton (1961): Nuclear Sizes (Oxford Univ. Press).
6. M. E. Grypeos (1969): Nuovo Cimento Letters 2, 117.
7. M. Harada, R. Tamagaki, H. Tanaka (1966): Progr. Theor. Phys., 35, 177.
8. A. Kallio, K. Koltveit (1964): Nucl. Phys., 53, 87.
9. S. E. Massen, M. E. Grypeos (1975): Nuovo Cimento, 30A, 1.
10. S. E. Massen, M. E. Grypeos: To be pul lished, Sci. Ann. Fac. Phys. aud Mathem., Univ. of Thessaloniki.
11. S. A. Moszkowski, B. L. Scott (1960): Ann. Phys., 11, 63.
12. T. Ohmura, M. Morita, Y. Yamada (1956): Progr. Theor. Phys., 15, 222.

ΠΕΡΙΛΗΨΗ

ΕΦΑΡΜΟΓΗ ΤΟΥ ΜΗ ΜΟΝΑΔΙΑΙΟΥ ΤΕΛΕΣΤΟΥ ΠΡΟΤΥΠΟΥ ΣΤΟΝ ΠΥΡΗΝΑ ^{16}O

Ὑπὸ
Σ. Η. ΜΑΣΕΝ

(Σπουδαστήριο Θεωρητικῆς Φυσικῆς, Πανεπιστήμιο Θεσσαλονίκης)

Ἡ μέθοδος τοῦ μὴ μοναδιαίου τελεστοῦ προτύπου, ποὺ ἔχει περιγραφεῖ σὲ προηγούμενη ἐργασία, ἐφαρμόζεται στὸν πυρήνα ^{16}O. Δίδονται δύο προσεγγιστικὲς ἐκφράσεις τῆς ἐνεργείας θεμελιώδους καταστάσεως τοῦ πυρήνα αὐτοῦ, χρησιμοποιώντας τὴν ἀρχὴ τῶν μεταβολῶν μὲ τὴν συνθήκη διαχωρισμοῦ. Στοὺς ὑπολογισμοὺς χρησιμοποιοῦνται ἁπλᾶ ρεαλιστικὰ δυναμικὰ σκληροῦ καὶ μαλακοῦ πυρήνα καὶ γίνεται συζήτηση τῶν ἀποτελεσμάτων ποὺ λαμβάνονται μὲ τὶς δύο μεθόδους.