

## DRAINAGE DENSITY EVOLUTION OVER TIME: A THEORETICAL APPROACH

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### ABSTRACT

In this paper we attempt a theoretical investigation of the drainage density evolution. It is assumed that the drainage density ( $D_s$ ) starts with a low values for a new geomorphic surface, it increase approaching a maximum value ( $D_{ss}$ ) and after that it decrease again to obtain very low values. The proposed model is based to the following differential equation :

$$dD_s/dt = a \cdot E(t) \cdot (D_{ss}-D_s/D_{ss}) - \beta \cdot D_s$$

where  $E(t)$  is a function of the erosional factor and  $a$  and  $b$  are constants depending if the physical properties of the geomorphic surface.

The values of the function  $E(t)$  is dependent on the Precipitation, the Lithology and the geomorphic stage of a given area.

### ΣΥΝΟΨΗ

Σ' αυτό το άρθρο επιχειρείται να γίνει μια θεωρητική εξέλιξη της υδρογραφικής πυκνότητας συναρτήσει του χρόνου. Εμείς υποθέτουμε ότι η υδρογραφική πυκνότητα για μία νέα γεωμορφική επιφάνεια αρχίζει από μικρές τιμές τείνει σε μία μέγιστη τιμή ( $D_{ss}$ ) και στη συνέχεια ελαττώνεται πάλι.

Το προτεινόμενο μοντέλο βασίζεται στη παρακάτω διαφορική εξίσωση

$$dD_s/dt = a \cdot E(t) \cdot (D_{ss}-D_s/D_{ss}) - \beta \cdot D_s$$

όπου  $E(t)$  είναι μία συνάρτηση που εξαρτάται από την διάβρωση και τα  $a$  και  $\beta$  είναι σταθερές που εξαρτώνται από τις φυσικές ιδιότητες της γεωμορφικής επιφάνειας.

### INTRODUCTION

Most of the geomorphological papers have emphasized in the statistical relationships between the variables of the drainage basin. However these relationships may be change during the time (Gregory and Walling, 1973). Drainage density it is one of the most important variable indicating the geomorphic stage of a region. It is interesting to investigate a long-term time variation of the drainage density over time, despite the fact that there is a total lack of empirical studies on this subject.

### THE PROPOSED MODEL

The main assumption of this paper is that the evolution of a drainage system

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involve different stages from "initiation" to maximum extension" and the gradual decline thereafter, due to the lateral abstraction of the streams. Such evolutionary scheme firstly proposed by Glock (1931). In this model the drainage density ( $D_s$ ) is depends of the value of saturation level of ( $D_{ss}$ ) at the drainage area and the erosional factor ( $E(t)$ ). By  $D_{ss}$  we mean the value of the upper limit of the drainage density. The erosional factor  $E(t)$  is dependent on the Precipitation, the Lithology and the geomorphic stage. To express the decline of the drainage density after the maximum extension at the drainage system a negative term is added ( $-\beta D_s$ ). Then we have a differential equation:

$$dD_s/dt = a \cdot E(t) \cdot (D_{ss}-D_s/D_{ss}) - \beta \cdot D_s \quad (1)$$

where  $a$  and  $\hat{a}$  are constants depending at the physical properties of the geomorphic surface.

The equation (1) after rearranging it becomes :

$$dD_s/dt + (a \cdot E(t)/D_{ss} + \beta) \cdot D_s = a \cdot E(t) \quad (2)$$

$$dD_s/dt + E'(t) \cdot D_s = b'(t) \quad (3)$$

$$E'(t) = a \cdot E(t)/D_{ss} + \beta$$

and putting

$$b'(t) = a \cdot E(t)$$

The equation (2) is nonhomogeneous.

The solution of the first-order linear differential equation, can be obtained by multiplying both sides of (3) by the function :

$$\mu(t) = \exp(\int E'(t) \cdot dt)$$

Hence, the general solution of (3) is :

$$D_s = 1/\mu(t) (\int \mu(t) b'(t) dt + c)$$

where  $c$  is the constant of integration.

If it is assumed that the function  $E(t)$  is constant  $E(t)=E$  over a specified time interval and takes zero value outside this interval  $E(t)=0$  then equation (1) becomes

$$dD_s/dt + (aE/D_{ss} + \beta) D_s = a \cdot E \quad (4) \text{ for } 0 < t < t_1 \text{ and}$$

$$dD_s/dt = -\beta \cdot D_s \quad (5) \text{ for } t > t_1$$

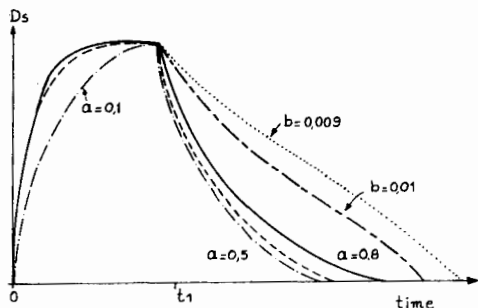
The solution of (4) is :

$$D_s = aE/k + (D_{s0} - aE/k) e^{-kt} \quad \text{where } aE/D_{ss} + \beta = k$$

giving that at  $t=0 \rightarrow D_s = D_{s0}$

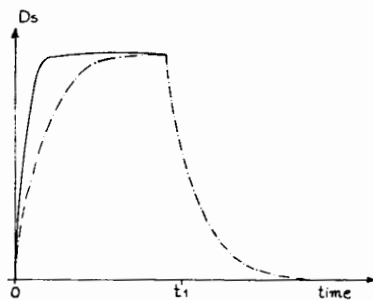
The solution of (5) is :

$$D_s = D_{s0} \cdot e^{-\beta t}$$



**Fig. 1:** Drainage density over time. For the time  $0 < t < t_1$  the erosional factor function is considered constant ( $E = \text{constant}$ ), for  $t > t_1$ ,  $E = 0$ .

**Εχ. 1:** Διάγραμμα πυκνότητας υδρογραφικού δικτύου συναρτήσει του χρόνου. Για την χρονική περίοδο  $0 < t < t_1$ , η τιμή του παράγοντα διάβρωσης ( $E$ ) θεωρείται σταθερά, για  $t > t_1$  το  $E = 0$ .



**Fig. 2:** Drainage density over time, for different values of  $E$ .

**Εχ. 2:** Διάγραμμα υδρογραφικής πυκνότητας συναρτήσει του χρόνου για διαφορετικές τιμές του  $E$ .

where  $D_{s0}$  is the value of  $D_s$  at the time  $t_1$ .

The combined solutions of (4) and (5) are sketched in Figures 1 and 2.

Figures 1,2 shows the change of drainage density ( $D_s$ ) over time. In Figure 1 the curves are desired by using  $D_{s0} = 1 \text{ km/km}^2$ ,  $D_{ss} = 50 \text{ km/km}^2$ .

$E = 5$  time units  $^{-1}$ ,  $a = \{0,1, 0,5, 0,8\}$  and  $\beta = \{0,025, 0,01, 0,009\}$ .

It must be noted that the form of the curve is changing as we change the parameters at the equations (4) and (5). For example as the values for the constant  $\beta$  are decreasing the rest of the curves after the time  $t_1$  tend to have low slopes, it means that the decay (erosion) of the whole relief is very slow. On the contrary at the beginning of the erosion the rate of growth of drainage density is very high until it achieves its maximum value ( $dD_s/dt = 0$ ).

In Figure 2 we change the value of the erosional factor  $E$ . It is obvious that the curve (a) with high erosional factor attains faster its maximum value.

## DISCUSSION

The drainage density variation over time reflects the whole relief evolution of a region. The proposed model suggests that in the absence of tectonic deformation a geomorphic surface will evolve in three main stages: firstly a relatively rapid growth of drainage density, second a period of steady values of drainage density and third a gradual decay of the above variable. The numerical solutions of the equations (4) and (5) and the resulted curves are very illustrative. However it is difficult to test the model because as already noted there is lack of appropriate data. There are a few papers such as of Ruhe (1952), Schum (1956), Leopold et al (1964) and Kashiwaya (1987) where there is an attempt to place drainage development in time sequence. The above sporadic data seems to apply to the present model.

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